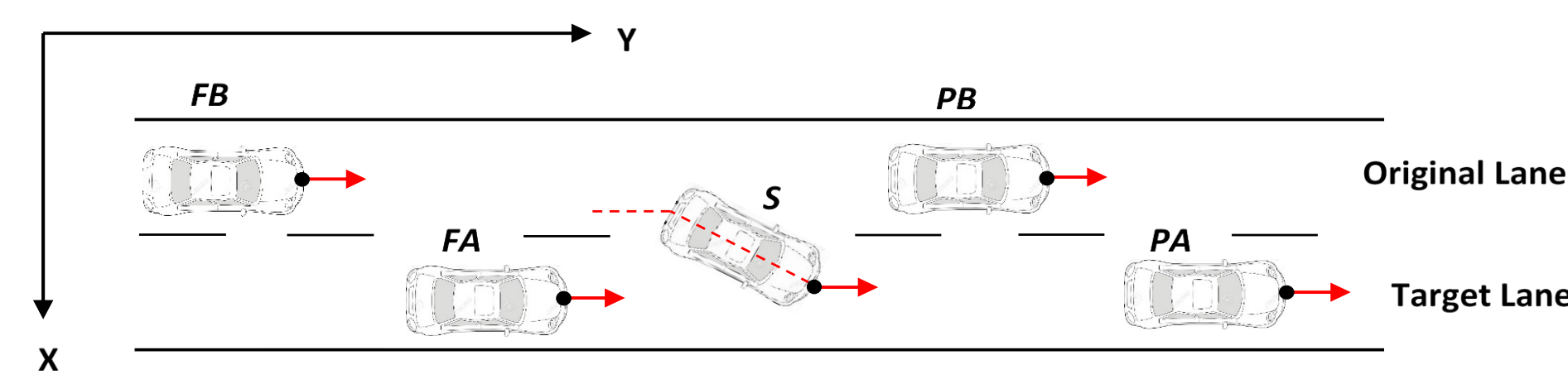


# Comparisons of Mandatory and Discretionary Lane Changing Behavior on Freeways

Matthew Vechione, Esmaeil Balal, and Ruey Long Cheu  
The University of Texas at El Paso

## Background

- A lane change is a lateral movement of a vehicle which is always accompanied with a longitudinal movement.
- A lane changing event involves up to five vehicles (see  $S, FB, PB, FA, PA$  in the figure below).



- A lane change may be modeled as a four-step process:
  - (1) motivation;
  - (2) selection of target lane;
  - (3) **checking for opportunity to move**; and
  - (4) the actual move.
 This research focuses on step (3).
- There are two types of lane changes on freeways: mandatory and discretionary.

- A Mandatory Lane Change (MLC) occurs when a driver **must** change lanes to exit a freeway, avoid a lane closure downstream, turn at a downstream intersection, etc.
- A Discretionary Lane Change (DLC) occurs at a driver's own **discretion** for faster speed, greater following distance, further line-of-sight, etc.

- A driver is expected to have different decision rules and/or risk-taking behavior for the two types of lane changes.

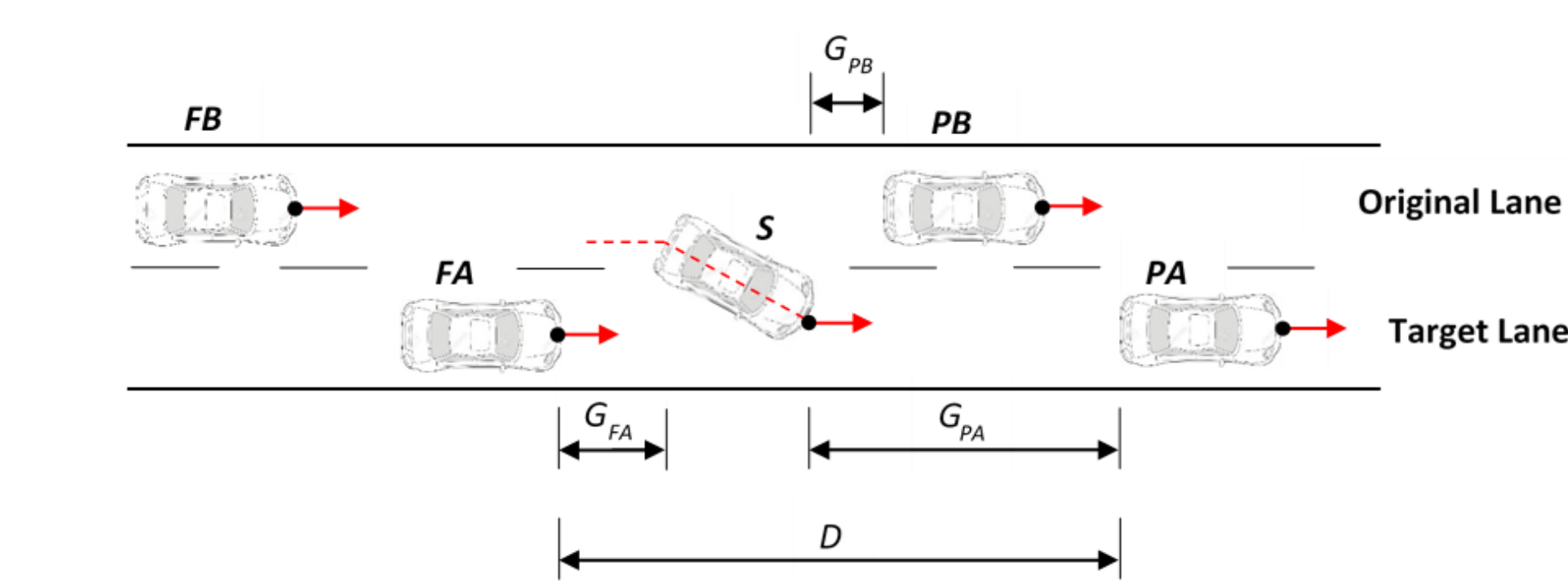
## Objectives

The objectives of this research are to:

- Examine descriptive statistics for variables that describe vehicle interactions for MLCs and DLCs, respectively.
- For each variable, conduct hypothesis test on the difference between the means of MLCs and DLCs.
- For each variable, apply the Kolmogorov-Smirnov (KS) test to test the difference in the *observed* cumulative probability distributions between MLCs and DLCs.
- For each variable, fit the probability distributions to the MLC and DLC data respectively, and use the KS test to test the difference between the *fitted* probability distributions.

## Literature Review

- Based on a survey from 443 drivers in El Paso, TX by Balal *et al.* (2014), the top four input parameters were gaps and distances, as shown in the figure below.



Notation	Definition	Unit	Range
$G_{PB}$	Gap between $S$ and $PB$	m	$\geq 0$
$G_{PA}$	Gap between $S$ and $PA$	m	$\geq 0$
$G_{FA}$	Gap between $S$ and $FA$	m	$\geq 0$
$D$	Gap between $PA$ and $FA$	m	$\geq 0$

- Formulas for calculating the lane changing variables are:

Front gap before lane change (in meters):

$$G_{PB} = (Y_{PB} - L_{PB}) - (Y_S), G_{PB} \geq 0$$

Front gap after lane change (in meters):

$$G_{PA} = (Y_{PA} - L_{PA}) - (Y_S), G_{PA} \geq 0$$

Rear gap after lane change (in meters):

$$G_{FA} = (Y_S - L_S) - (Y_{FA}), G_{FA} \geq 0$$

Distance (in meters):

$$D = (Y_{PA} - L_{PA}) - (Y_{FA}), D \geq 0$$

where:

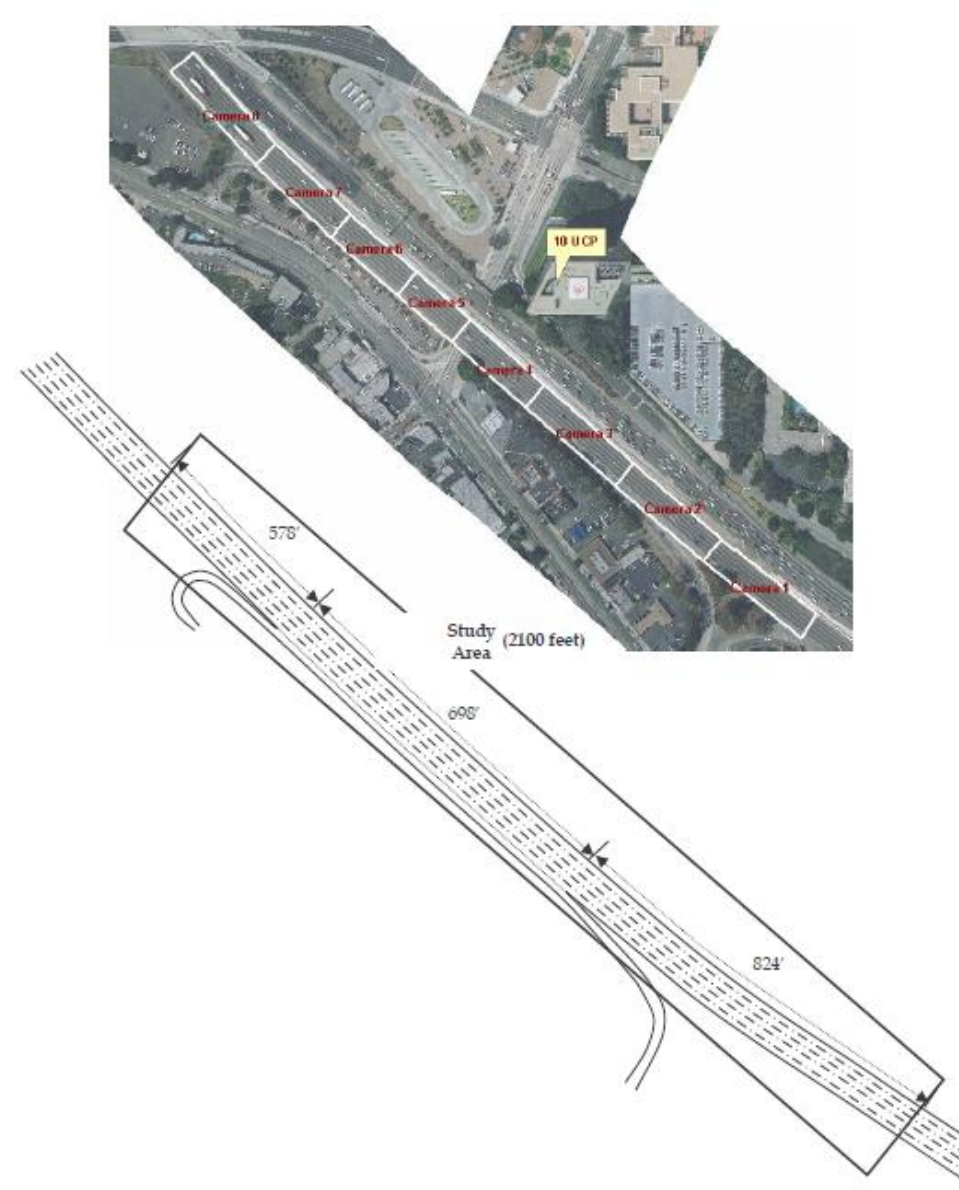
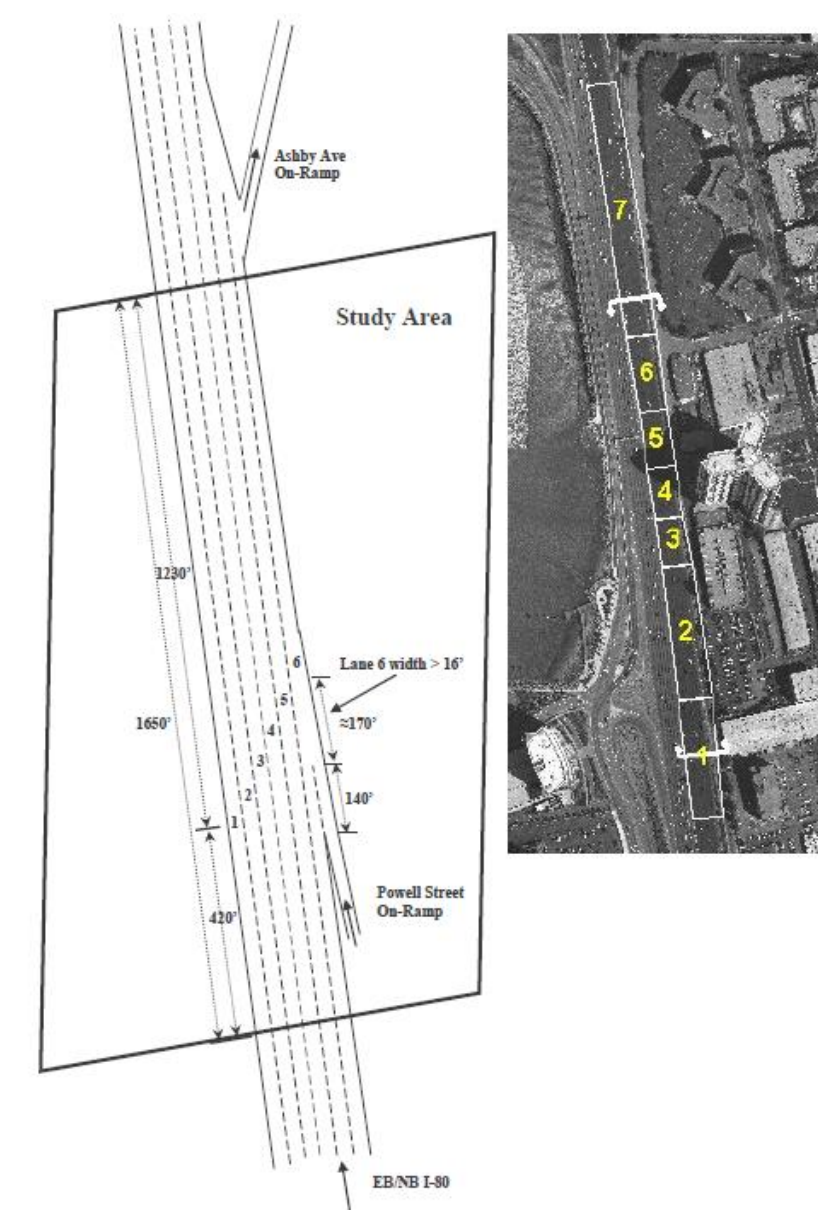
- $L$  is the length of the vehicle;
- $Y$  is the longitudinal position of each vehicle;
- $P$  represents a preceding vehicle;
- $F$  represents a following vehicle;
- $B$  is before the lane change; and
- $A$  is after the lane change.

## Vehicle Trajectory Data

- From NGSIM data base.

Interstate 80 (I-80)  
Emeryville, CA  
**Dataset A**

U.S. Highway 101  
Los Angeles, CA  
**Dataset B**



Cambridge Systematics, Inc. (2005)

## Methodology

- Only passenger cars selected as subject vehicles;
- Vehicles that changed lanes between lanes 5 and 6 were assumed to make a MLC;
- Vehicles that changed lanes between lanes 2 to lane 4 were assumed to make a DLC;
- Lane 1 omitted, as it is a HOV lane;
- For each subject vehicle, the time  $t$  when the lane changing event occurred was taken as time when the front center of the subject vehicle crossed the lane markers;
- Variable values were calculated at  $t-0.4, t-0.3, t-0.2, t-0.1$ , and  $t$  seconds, and the average values from  $t-0.4$  to  $t$  seconds were used as the representative value. The averaging of data to 0.5 second intervals was to:
  - Reduce error caused by instantaneous values in the NGSIM data;
  - Be more consistent with human perception time; and
  - Be consistent with other research that used NGSIM data.

## Statistical Analyses

### 1. Descriptive Statistics

- Dataset A

Variable	$G_{PB}$		$G_{PA}$		$G_{FA}$		$D$	
	MLC	DLC	MLC	DLC	MLC	DLC	MLC	DLC
Unit	m		m		m		m	
Sample size	166	135	166	135	166	135	166	135
Min	0.61	4.33	0.31	0.07	1.40	0.49	10.25	6.06
Max	124.26	76.97	47.75	105.37	80.03	93.07	115.33	162.15
Mean	15.08	15.18	10.32	11.46	15.35	17.58	30.12	33.42
Std. deviation	13.97	8.63	8.66	13.43	12.10	14.66	16.65	20.85
Skewness	4.07	3.13	1.97	3.44	2.11	1.94	1.91	2.65

- Dataset B

Variable	$G_{PB}$		$G_{PA}$		$G_{FA}$		$D$	
	MLC	DLC	MLC	DLC	MLC	DLC	MLC	DLC
Unit	m		m		m		m	
Sample size	71	128	71	128	71	128	71	128
Min	5.63	3.79	3.46	0.82	1.93	0.45	11.51	15.24
Max	185.81	74.08	160.89	216.07	92.06	103.51	172.54	234.74
Mean	50.74	19.11	22.44	20.83	22.88	20.95	49.72	46.06
Std. deviation	40.70	12.72	24.01	24.82	18.69	16.10	30.80	27.49
Skewness	1.11	2.06	3.35	4.65	1.59	2.37	1.64	3.34

### 2. Difference Between Two Means

- Dataset A

Variable	$G_{PB}$		$G_{PA}$		$G_{FA}$		$D$	
	MLC	DLC	MLC	DLC	MLC	DLC	MLC	DLC
Unit	m		m		m		m	
Sample size	166	135	166	135	166	135	166	135
Mean	15.08	15.18	10.32	11.46	15.35	17.58	30.12	33.42
Std. deviation	13.97	8.63	8.66	13.43	12.10	14.66	16.65	20.85
t-statistic	-0.07		-0.85		-1.42		-1.49	
p-value	0.944		0.398		0.157		0.137	
Conclusion ( $\alpha=0.025$ )	Fail to reject $H_0$		Fail to reject $H_0$		Fail to reject $H_0$		Fail to reject $H_0$	

- Dataset B

Variable	$G_{PB}$		$G_{PA}$		$G_{FA}$		$D$	
	MLC	DLC	MLC	DLC	MLC	DLC	MLC	DLC
Unit	m		m		m		m	
Sample size	71	128	71	128	71	128	71	128
Mean	50.74	19.11	22.44	20.83	22.88	20.95	49.72	46.06
Std. deviation	40.70	12.72	24.01	24.82	18.69	16.10	30.80	27.49
t-statistic	6.38		0.45		0.73		0.83	
p-value	0.000		0.654		0.466		0.406	
Conclusion ( $\alpha=0.025$ )	<b>Reject <math>H_0</math></b>		Fail to reject $H_0$		Fail to reject $H_0$		Fail to reject $H_0$	

### 3. Observed Distributions

- The KS test compares a *cumulative distribution* against the theoretical cumulative distribution or two *cumulative distributions* against one another.
- The maximum difference between the two distributions is computed by:

$$d = \max_{x,y} |F(x) - F(y)|$$

- $d$  is compared to a critical value  $d_{n_1, n_2, \alpha}$ :

$$d_{n_1, n_2, \alpha} = k_\alpha \sqrt{\frac{n_1 + n_2}{n_1 n_2}}$$

where  $n_1$  and  $n_2$  are the sample sizes of the two distributions,  $k_\alpha$  is the KS test parameter with level of significance =  $\alpha$ .

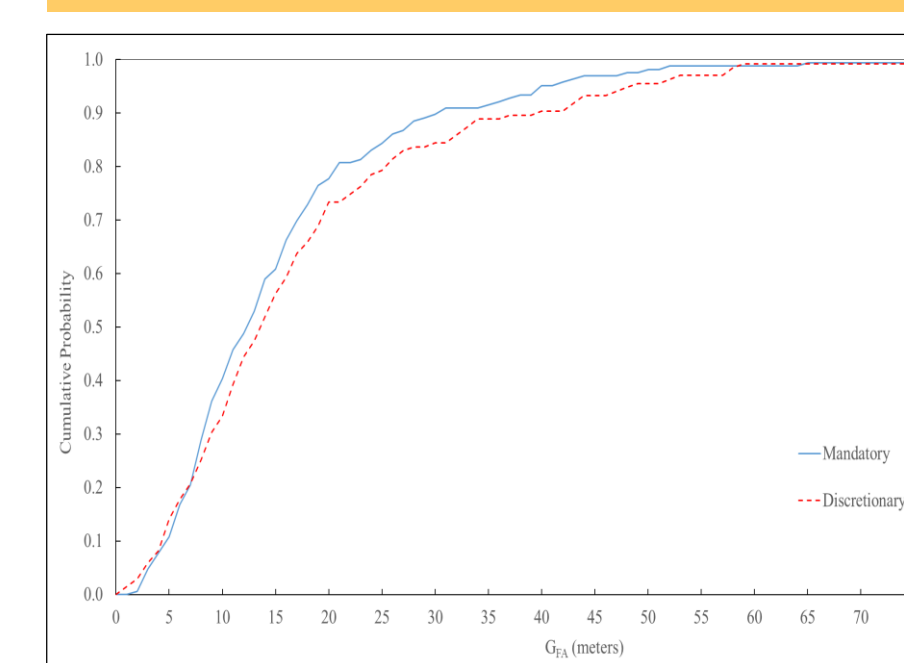
- Dataset A

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
$d$	0.164	0.126	0.076	0.085
$\alpha=0.10$ Critical Value = 0.142	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
$\alpha=0.05$ Critical Value = 0.157	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$

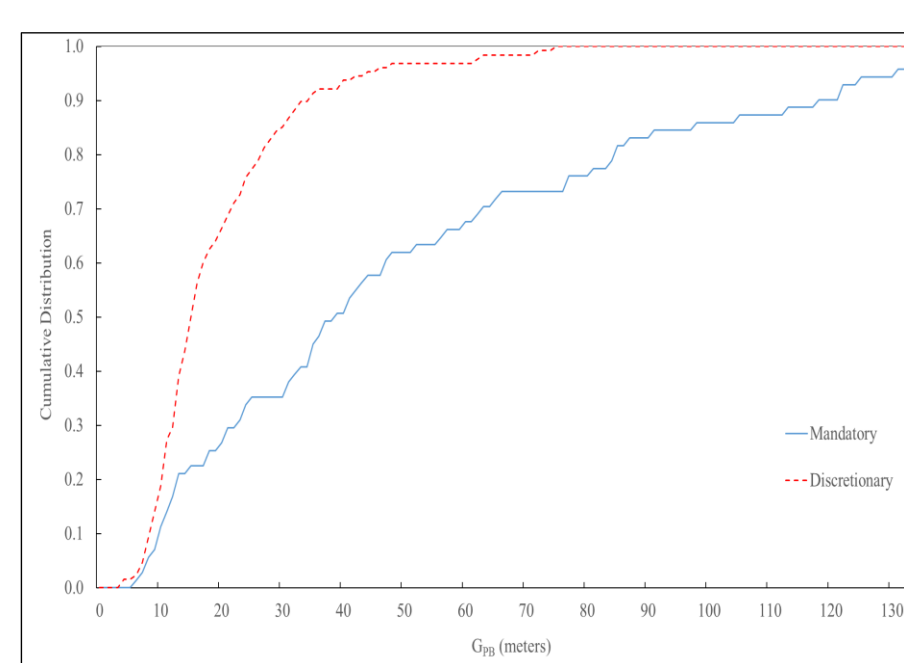
- Dataset B

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
$d$	0.499	0.101	0.108	0.146
$\alpha=0.10$ Critical Value = 0.181	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
$\alpha=0.05$ Critical Value = 0.201	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$

Best case ( $G_{FA}$  in Dataset A)



Worst case ( $G_{PB}$  in Dataset B)



### 4. Fitted Probability Distributions

- The observed data was then fitted with probability distributions using @RISK.

- MLC

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
Dataset A				
Unit	m	m	m	m
Best fit	Log-logistic	Pearson 5	<b>Log-normal</b>	Gamma
2 <sup>nd</sup> best fit	Pearson 5	<b>Log-normal</b>	Inverse Gaussian	Inverse Gaussian
3 <sup>rd</sup> best fit	<b>Log-normal</b>	Inverse Gaussian	Pearson 5	<b>Log-normal</b>
Recommended	Log-normal			
Log-normal location parameter, $\lambda$	2.404	2.068	2.489	3.272
Log-normal scale parameter, $\xi$	0.787	0.730	0.695	0.516

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
Dataset B				
Unit	m	m	m	m
Best fit	Exponential	Inverse Gaussian	Inverse Gaussian	Inverse Gaussian
2 <sup>nd</sup> best fit	Inverse Gaussian	<b>Log-normal</b>	<b>Log-normal</b>	Gamma
3 <sup>rd</sup> best fit	<b>Log-normal</b>	Pearson 5	Pearson 5	<b>Log-normal</b>
Recommended	Log-normal			
Log-normal location parameter, $\lambda$	3.678	2.729	2.874	3.744
Log-normal scale parameter, $\xi$	0.705	0.873	0.715	0.570

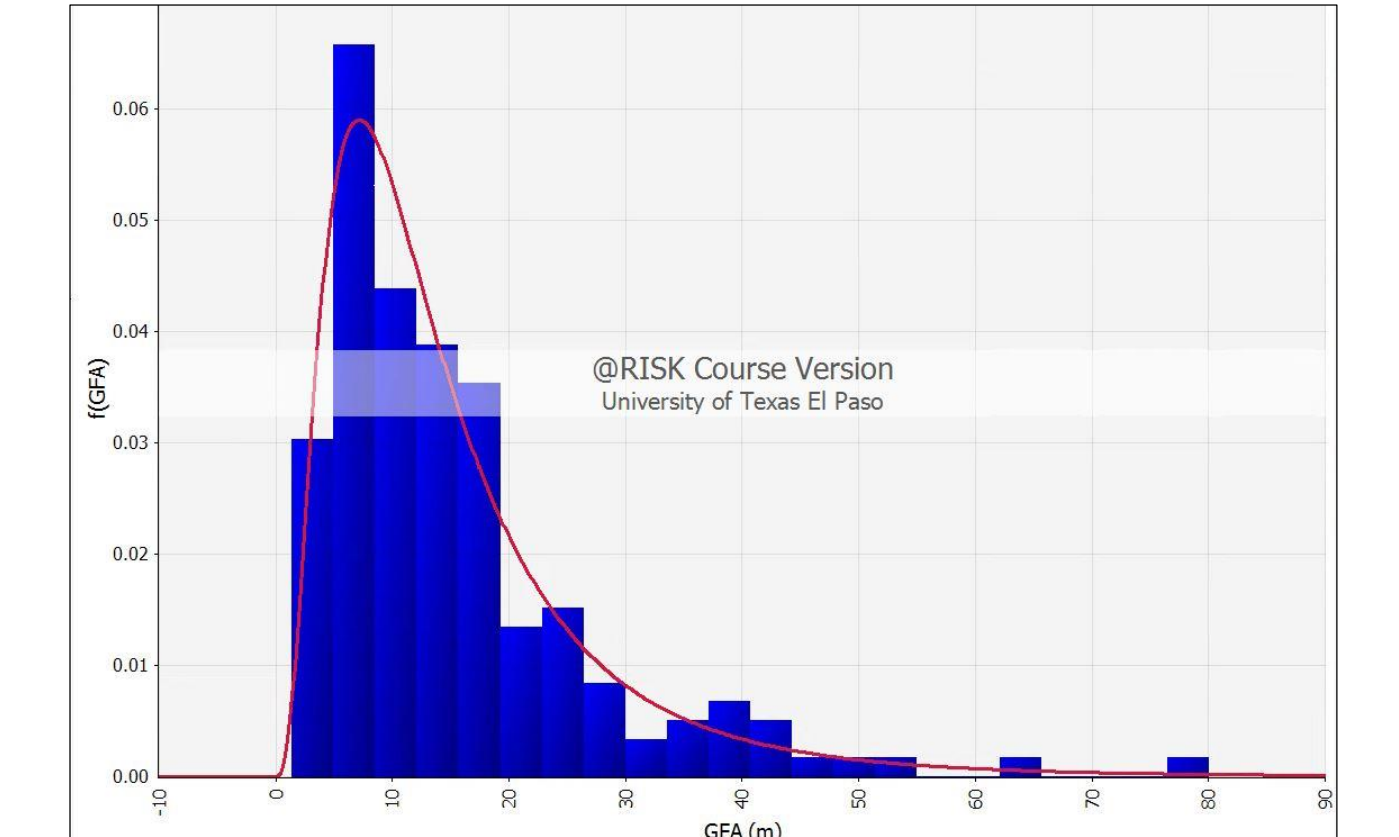
- DLC:

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
Dataset A				
Unit	m	m	m	m
Best fit	Pearson 5	Exponential	Inverse Gaussian	Log-logistic
2 <sup>nd</sup> best fit	Log-logistic	<b>Log-normal</b>	<b>Log-normal</b>	Pearson 5
3 <sup>rd</sup> best fit	<b>Log-normal</b>	Inverse Gaussian	Pearson 5	<b>Log-normal</b>
Recommended	Log-normal			
Log-normal location parameter, $\lambda$	2.580	2.006	2.603	3.345
Log-normal scale parameter, $\xi$	0.529	0.930	0.726	0.573

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
Dataset B				
Unit	m	m	m	m
Best fit	Pearson 5	<b>Log-normal</b>	Log-logistic	Log-logistic
2 <sup>nd</sup> best fit	<b>Log-normal</b>	Log-logistic	Pearson 5	Pearson 5
3 <sup>rd</sup> best fit	Log-logistic	Pearson 5	<b>Log-normal</b>	<b>Log-normal</b>
Recommended	Log-normal			
Log-normal location parameter, $\lambda$	2.767	2.594	2.810	3.678
Log-normal scale parameter, $\xi$	0.606	0.940	0.681	0.552

- The lognormal distribution was recommended.

- Example: log-normal distribution fitted for  $G_{FA}$ , Dataset A:



- The KS test was then applied to the fitted log-normal distributions for MLC and DLC.

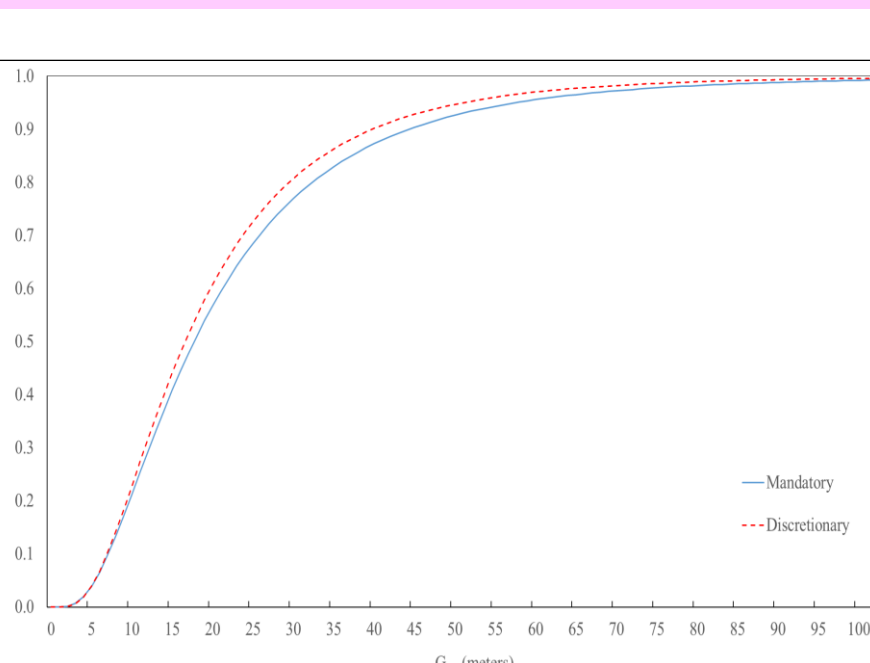
- Dataset A - difference between MLC and DLC

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
$d$	0.168	0.077	0.066	0.066
Critical Value	0.155	0.165	0.178	0.135
Conclusion	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
$\alpha=0.10$				
Critical Value	0.172	0.183	0.197	0.150
Conclusion	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
$\alpha=0.05$				
Critical Value	0.187	0.197	0.212	0.165
Conclusion	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$

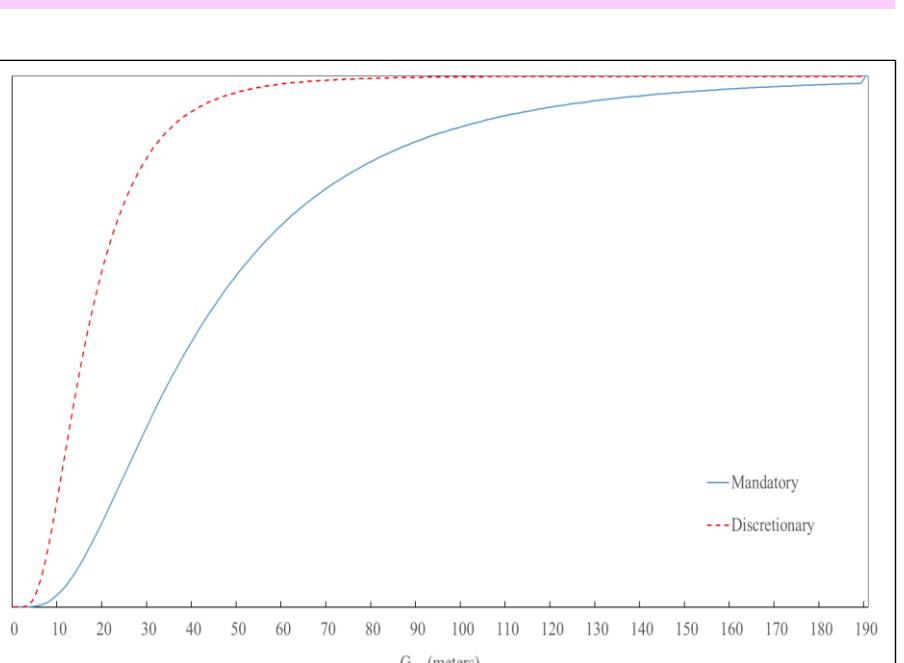
- Dataset B - difference between MLC and DLC

Variable	$G_{PB}$	$G_{PA}$	$G_{FA}$	$D$
$d$	0.515	0.066	0.041	0.049
Critical Value	0.126	0.117	0.169	0.113
Conclusion	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
$\alpha=0.10$				
Critical Value	0.139	0.129	0.187	0.125
Conclusion	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$
$\alpha=0.05$				
Critical Value	0.152	0.142	0.199	0.140
Conclusion	<b>Reject <math>H_0</math></b>	Fail to reject $H_0$	Fail to reject $H_0$	Fail to reject $H_0$

Best case ( $G_{FA}$  in Dataset B)



Worst case ( $G_{PB}$  in Dataset B)



## Conclusions

- All variables may be described by the log-normal distribution.
- There is no significant difference between MLC and DLC for the three variables in the **target lane** (i.e.  $G_{PA}, G_{FA}$ , and  $D$ ).
  - These may be common variables between MLCs and DLCs
- For  $G_{PB}$  (in the original lane), significant differences are found between MLC and DLC:
  - Population means between MLCs and DLCs in