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Traffic Signal Optimization and Coordination in Connected Cities

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16. Abstract

The major tasksof this project include:

1. Development of CV-based traffic signal timing optimization methods utilizing individual vehicles' trajectories (i.e., second-by-second vehicle locations and speeds). This includes methods for timing plan optimization (of a single intersection) and coordination optimization among multiple intersections. The proposed method evaluates the total weighted sum of travel times and fuel consumptions of all vehicles in the study area in the optimal green time and offset determination.
2. Propose solution methods for CV-based traffic signal optimization, which includes a DP with two-step method for intersection level optimization (phase duration optimization) and a prediction-based solution method for the two-level problem (offset optimization) under corridor level optimization.
3. Comprehensive testing and validation of the proposed methods in traffic simulation. Various combinations of travel demands and types of CVs are tested for the proposed signal timing optimization methods. The testing tasks should validate that the developed methods are computationally manageable and have the potential to be implemented in CV-based traffic signal applications in the real world.

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Traffic Signal Optimization and Coordination in Connected Cities

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Executive Summary

Connected vehicles in smart cities, including vehicle to vehicle (V2V), vehicle to infrastructure (V2I), and vehicle to anything (V2X) communications, can provide more opportunities and impose more challenges for urban traffic signal control. This project aims to develop a framework, including modeling techniques, algorithms, and testing strategies, for urban traffic signal optimization with CVs. This framework is able to optimize traffic signal timing for a single intersection or along a corridor. More specifically, the major tasks of this project include:

1. Development of CV-based traffic signal timing optimization methods utilizing individual vehicles' trajectories (i.e., second-by-second vehicle locations and speeds). This includes methods for timing plan optimization (of a single intersection) and coordination optimization among multiple intersections. The proposed method evaluates the total weighted sum of travel times and fuel consumptions of all vehicles in the study area in the optimal green time and offset determination.
2. Propose solution methods for CV-based traffic signal optimization, which includes a DP with two-step method for intersection level optimization (phase duration optimization) and a prediction-based solution method for the two-level problem (offset optimization) under corridor level optimization.
3. Comprehensive testing and validation of the proposed methods in traffic simulation. Various combinations of travel demands and types of CVs are tested for the proposed signal timing optimization methods. The testing tasks should validate that the developed methods are computationally manageable and have the potential to be implemented in CV-based traffic signal applications in the real world.

Future work may also investigate how different penetrations of CV-equipped vehicles will affect the performance of the proposed signal control method. This will require estimating the trajectories of vehicles that are not equipped with CV technology. When sample trajectory data from the real world are available, certain stochastic methods may be applied to estimate and predict vehicle trajectories. Furthermore, the proposed method needs to be tested using real world traffic signals and CV data.

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1. Introduction

1.1 Motivation

1.1.1 Urban congestion and challenge of traffic signal control

As a critical infrastructure that is crucial to the economy and the daily life of everyone, transportation also creates severe congestion and consumes tremendous energy. In the United States, the gasoline consumption by the transportation sector was about 143.37 billion gallons in 2016, a daily average of about 9.33 million barrels[1]. At the same time, traffic congestion on urban roads causes extra fuel consumption as well as additional travel delays. The 2015 Urban Mobility Scorecard[2] estimated that U.S. highway congestion costs $160 billion a year, and an average American commuter loses 42 hours per year due to traffic congestion. Therefore, it is imperative to reduce traffic delays and improve transportation energy efficiency in urban areas.

Previously, most traffic signal researchers assumed that infrastructure sensors (such as loop detectors or video cameras) were the major source of information on traffic conditions. Traffic control systems mainly relied on manually collected traffic counts and data from infrastructure sensors. Traffic signal plans were developed based on arrival vehicles adjusted by the time of day. However, point detectors and video detectors have many disadvantages. Point detectors only record the location of vehicles when they pass by. There is no trajectories information, such as speeds, positions, and accelerations of a vehicle. Detectors at stop bars have higher failure rates because of the rigorous vehicle braking and accelerating behaviors[3-4]. In addition, maintenance of the detector is time consuming and costly. The performance of video detectors could be negatively impacted by environmental conditions, such as lighting (the most cited condition causing video detector failure) and weather[5]. These limitations of detectors can be significantly improved by more advanced data sources.

1.1.2 Connected vehicle and V2X technologies

Instead of relying on infrastructure sensors such as loop detectors, urban traffic signal control can be transformed by Connected Vehicle (CV) technology. CV enables vehicle-to-everything (V2X) communications and leads to an intelligent transportation system where all vehicles, road users, and infrastructure systems can communicate with each other. Various communication technologies can be applied, such as cellular, Wi-Fi, satellite radio, or dedicated short-range communication (DSRC)[6]. A summary of the pros and cons of different communication methods is presented in Table 1. Although cellular networks cover the majority of the locations where people live and work, there are areas where cellular service is not available. Long-term evolution (LTE) is a promising technology that can help deliver data more quickly. However, the transmission rate is a major issue when users are moving or in an area with many other LTE users. A newer and faster 5G network will allow instantaneous data transmission

rates, which enables new technologies like CVs. At the initial stages, 5G networks will be expensive for carriers and may only cover a small number of users. Privacy and high cost of cellular data are other concerns for cellular communications. Wi-Fi technology offers higher data rates, but it has similar cost and security concerns to cellular communications. Satellite radios have the disadvantage of slow download time for satellite communication. DSRC is a mature communication technology that ensures reliable and secure communications when vehicles are operating at high speeds. Cost and security risks are the main concerns for DSRC technology.

Communication methods	Pros	Cons
Cellular	1. Offer widespread coverage throughout the nation ; 2. Long-term evolution (LTE) delivers data quickly. 3. 5G network provides more bandwidth for everyone	1. Dead spots exist (area cellular services are not available); 2. Transmission rates slow down when user is moving or in a area with many other LTE users; 3. Security risks 4. High cost of cellular data 5. Small coverage of 5G network.
Wi-Fi	1. Offers Higher data rates	1. Slow transmission rates if a user is moving 2. Security cost 3. Price concerns
Satellite Radio	1. Provide broadcast service national wide	1. Not covering Alaska and Hawaii 2. Data download time is slow 3. Security risks
DSRC	1. Provides instantaneous network Connectivity and message transmission 2. Has a designated licensed bandwidth to permit secure reliable communication 3. Provides high data transmission rates	1. Security risks 2. Cost concerns

Table 1: Pros and cons of different communication methods

CV/V2X will provide more information about traffic conditions, which in turn will help reduce congestion, reduce accident rates, maximize traffic flows and minimize emissions. With Vehicle-to-Vehicle (V2V) communications, vehicle position, speed, acceleration, etc. can be exchanged among nearby vehicles. With Vehicle-to-Infrastructure (V2I) communications, vehicles can communicate to traffic signals, work zones, tollbooths, and other types of infrastructure to exchange information such as vehicle trajectories, traffic conditions, and signal timing, among others. Such information can be collected into “Basic Safety Message” (BSM) and other types of messages[7]. The information/data exchange among vehicles and between vehicles and the infrastructure has the potential to improve traffic mobility and safety, warn drivers of upcoming road conditions, and adjust traffic signal timing more efficiently at signalized intersections. For example, in 2011, Japan deployed the ITS Spot system to implement V2I on both local

roads and expressways by providing three services to drivers: dynamic route guidance, safe driving support, and electronic toll collection[8]. In 2013, Germany, the Netherlands and Austria worked on the deployment of a European Cooperative ITS (C-ITS) corridor that incorporates V2I to provide traveler information on roadwork and upcoming traffic[9]. In the United States, USDOT, transportation agencies, academic researchers and various stakeholders are engaged in the development of technologies and systems that enable V2V and V2I applications. From 2012 to 2014, USDOT deployed V2V DSRC devices on real roads with real drivers and evaluated the functional feasibility of V2V in Model Deployment in Ann Arbor, Michigan. There are approximately 2800 equipped vehicles, including cars, trucks, and transit. Overall, the experiment was successful in creating interactions between DSRC-equipped vehicles that can successfully communicate with each other. In the past decade, USDOT provided more than 600 million in funding for CV technologies. Over the next few years, USDOT plans to provide up to $100 million in funding for a number of pilot projects comprised of V2V and V2I technologies and applications[8].

The NHTSA is proposing a mandate to require all new light vehicles to be capable of V2V communications by 2022 so that 60% of vehicles (about 146 million) will be equipped with V2X/DSRC devices by 2029[6]. Similarly, the American Association of State Highway and Transportation Officials (AASHTO) predicted that 90% of light vehicles would be equipped with V2V technologies by 2040. AASHTO also estimated that by 2025, 20% of signalized intersections will be capable of V2I communication, and by 2040, 80% of signalized intersection will be V2I capable. Figure 1 shows the DOT's planned CV path to deployment from 2010 to 2040[8].

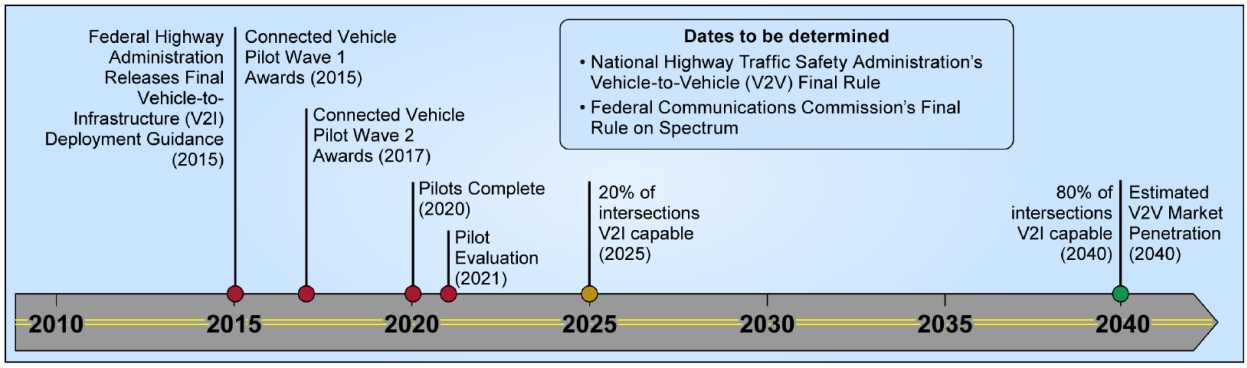


Figure 1: DOT's Planned Connected Vehicle Path to Deployment, 2010-2014[8]

The advent of CV technologies offers an opportunity to significantly enhance the transportation system, primarily in terms of improved safety. Communications between CVs can issue warnings before a potential crash, potentially reducing fatalities and serious injuries. According to the National Highway Traffic Safety Administration[6], as CV penetration and adoption of V2X technologies and safety related applications increase, 439,000 to 615,000 crashes, or about 13% to 18% of total light vehicle crashes, can be prevented annually by 2040. In addition, mobility and emission benefits will also likely emerge by taking advantage of CV technology. With V2I and V2V communications, vehicles approaching an intersection from different

directions can communicate with each other and with traffic infrastructure, which will enable the optimization of signal timing to reduce delay and fuel consumption. Emission benefits can also be achieved by providing feedback to the drivers on how to operate their vehicles at the most fuel-efficient states under different driving situations.

1.1.3 CV–based traffic signal control

Traffic signal control systems are the primary tools for urban traffic flow management on arterials, with the objective being to increase safety, improve traffic flow, and reduce traffic delays and fuel consumption. Over the past few decades, extensive efforts have been made to improve the efficiency of traffic signal control systems in order to alleviate ever-growing traffic demands. CV technologies that include V2V, V2I, and V2X communications have received increasing attention in signal timing studies. V2V/V2I communications bring new paradigms compared to traditional traffic signal operations. Traffic controllers can collect real-time data from CVs (position, speed, fuel consumption parameters), then process the data to optimize signal-timing plans at an intersection, along a corridor, or for a region in order to minimize delay, number of stops, and environmental impacts.

1.2 Objective

The advent and deployment of CV/V2X communications offer the potential to significantly improve the efficiency of traffic signal control systems. The knowledge of vehicle trajectories in the network allows for optimal signal setting and significant improvements in network performance compared to existing traffic signal control systems. The goal of this project is to investigate traffic signal timing optimization techniques based on CV data, i.e., real-time information on vehicles' locations and speeds, as well as communications to the signal control systems. With such information, the performance measurements can be defined, which allows fuel consumptions and travel times of all vehicles in the network to be estimated accurately and signal timing optimization strategies to be developed to optimize those measures. Ideally, with the full penetration rates of CVs, it is possible to know the complete states of traffic flows and predict how they are impacted by traffic signal settings.

In summary, the specific objectives of this project are:

1. Develop and evaluate signal timing optimization methods for isolated intersections under various penetration rates of CVs. By assuming a fixed cycle length (to facilitate signal coordination when optimizing multiple signals in later objectives), a mixed integer nonlinear programming (MINLP) that considers trajectories of individual vehicles is developed to minimize the total weighted sum of fuel consumptions and travel times of all vehicles in the study area. The MINLP model is approximated to a dynamic programming (DP) formulation to improve computational efficiency.

2. Develop coordinated signal operation schemes along corridors or for a network to optimize the offsets of multiple intersections and evaluate how each signal change affects the signal timing of nearby intersections. This involves the coordination of vehicles to provide a smooth propagation of the platoon on the arterials.
3. Testing and validating the models and algorithms developed in this project. The proposed signal timing optimization methods will be tested and validated using traffic simulation. The methods will be tested and compared against traditional fixed time and actuated signal control strategies.

1.3 Contributions

This project aims to develop a framework, including modeling techniques, algorithms, and testing strategies, for urban traffic signal optimization with CVs. This framework should be able to optimize traffic signal timing for a single intersection, along a corridor, or for a network. More specifically, the major contributions of this project include:

4. Development of CV-based traffic signal timing optimization methods utilizing individual vehicles' trajectories (i.e., second-by-second vehicle locations and speeds). This includes methods for timing plan optimization (of a single intersection) and coordination optimization among multiple intersections. The proposed method evaluates the total weighted sum of travel times and fuel consumptions of all vehicles in the study area in the optimal green time and offset determination.

5. Propose solution methods for CV-based traffic signal optimization that includes a DP with two-step method for intersection level optimization (phase duration optimization) and a prediction-based solution method for the two-level problem (offset optimization) under corridor level optimization.

6. Comprehensive testing and validation of the proposed methods in traffic simulation. Various combinations of travel demands and types of CV are tested for the proposed signal timing optimization methods. The testing tasks should validate that the developed methods are computationally manageable and have the potential to be implemented in CV-based traffic signal applications in the real world.

2. Literature Review

2.1 Traditional traffic signal control

Traditional traffic signal control problems have been extensively investigated, with a variety of methods tested, such as fixed-time control, actuated control, and adaptive control[10]. To ensure safety, most existing traffic signal control methods (at least in the US) are based on the dual ring design scheme, as shown in Figure 2. The dual ring scheme separates conflicting movements from different approaches, divides a cycle into phases, and determines the timing of each phase at an intersection. There are four movements for straight and/or right turn movements (movement number 2, 4, 6, and 8 in Figure 2) and four movements for left-turn movements (movement number 1, 3, 5, and 7 in Figure 2). The barriers or phase concurrency groups defines the conflicts between movements. In this research, the dual ring scheme is also applied to ensure traffic safety.

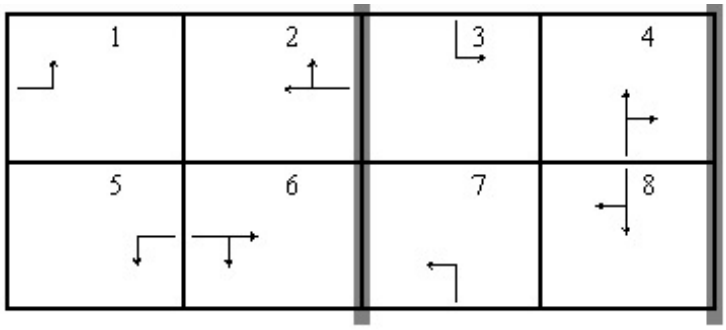


Figure 2: Dual ring diagram. Standard NEMA phasing[10]

There are three types of traffic controllers widely deployed all over the world: fixed-time controller, actuated traffic controller, and adaptive traffic controller. In the case of fixed-time traffic control, the signal timing variables (e.g., cycle length, split, and offset) are pre-determined based on the historical data. Different programs can be generated to accommodate various traffic demands, such as morning and evening peak, according to the time of day. However, such a system is rigid and cannot adapt to real-time fluctuations of vehicle arrivals, leading to inefficient and unsatisfied behaviors in many situations.

Actuated traffic control utilizes real time traffic states provided by loop detectors or other infrastructure sensors built upstream of the stop line to detect the vehicle arrivals. It usually maintains a green signal on the busiest street until a pedestrian or a vehicle on the less traveled side street approaches the intersection. The green time will be extended if there is a coming vehicle being detected. This system performs much better than the fixed time traffic control due to the following features: A phase can be skipped if there is no waiting vehicle. It can also be realized earlier if there is no demand from the conflicting movements. The green time can be extended to the maximum green split and can also be terminated if

there is a prioritized vehicle. In most cases, the signal plan in actuated traffic control is cycle-based with a pre-determined fixed signal sequence.

Adaptive signal control is the most advanced traffic signal control method so far. It still uses detection data like actuated traffic control, but it retrieves current traffic information and applies prediction models to forecast vehicle states, vehicle arrivals, and queue length in the near future. It continuously adjusts when green lights start and end based on the current traffic conditions, demand, and system capacity to accommodate traffic patterns to promote smooth flow and reduce traffic congestion. The most widely known systems include SCOOT[11] and SCATS[12]. These traffic signal controllers calculate signal plans based on traffic flow information over a look-ahead horizon. The look-ahead search algorithm with short-term prediction (e.g., less than one cycle) periods often leads to side effects, such as undeserving left-turns. In addition, this type of algorithm reevaluates decisions too infrequently (e.g. 4-5s) and are unable to terminate phases immediately when queues disperse earlier than predicted[13]. The look-ahead search algorithm is the foundation for some other adaptive traffic control methods, such as OPAC and COP. OPAC is a method for demand-responsive decentralized traffic signal control that requires on-line data from upstream approach detectors and from adjacent intersections. There is no coordination feature imbedded, but this method has self-coordination capabilities[14]. OPAC III implements a “rolling horizon” strategy to minimize stops and delays. They can respond to the variation of traffic flow since they emphasize traffic prediction. OPAC IV is a network version of OPAC. With the dramatic improvement of the computational capacity and advanced information collection and communication techniques, it is possible to utilize a large volume of real time data with OPAC IV. Recent adaptive control related research includes the swarm algorithm[15], platoon-based algorithms[16], rolling horizon approaches[17], oversaturation algorithm[18] and reinforcement-learning algorithm[19], among others. A comprehensive discussion of the signal control algorithms can be found in Goodall[20].

2.2 Coordination in traffic signal control

Traffic signal coordination can provide efficient movements of vehicle platoons through adjacent intersections and reduce travel times, delays and the number of stops. It is widely implemented for arterials, downtown areas, and closely spaced intersections. There are three parameters in coordination concepts: cycle length, splits, and offset. Cycle length is the total time to finish a complete sequence of all signal phases. For coordinated traffic signals, they normally need to have the same cycle length, called the *common* cycle length. In practice, such a common cycle length may be determined by signal design tools for coordination systems, such as Synchro and TRANSYT. The split is the sum of the green, yellow and all red intervals, which is the segment of the cycle length that allocated to each phase. The offset is the time difference between a fixed point in the cycle and a system reference point.

Coordination is not beneficial for all systems. It requires that the intersections be close to each other and that the traffic demands between the adjacent intersections be large. A Federal Highway Administration

(FHWA) report suggested that if the intersections are spaced within a certain distance (i.e., 0.75 miles), coordination can be considered[21]. If the arrival vehicles are random and unrelated to the operation of the upstream intersections, coordination may provide limited benefits.

Different signal controllers have different mechanisms to realize signal coordination. In the case of fixed time control, (i.e., downtown closely spaced intersections), traffic signal coordination is achieved by setting an appropriate offset value, which is the time difference of the fixed points in a cycle between the local intersection and master intersection. It requires the same cycle length for all coordinated intersections. For actuated traffic signal control, the cycle length also needs to be the same for the coordinated actuated systems. Wardberg *et al.*[22] suggested that coordination cannot be realized without the common cycle time of the whole systems. Coordination for actuated traffic signal can synchronize multiple intersections using force-off mode. Force-off is a point in a cycle where a phase must end. It ensures the coordinated phases are provided with a minimum amount of green time to implement the green wave. The uncoordinated phases either use the unused time of previous phases in fixed force-off mode or limit to their defined split amount of time in floating force-off mode[23]. Coordination for adaptive traffic signal can be achieved based on the common cycle, such as SCOOT system and RHODES system. Some multi agent systems (decentralized systems that focus on individual intersections) do not use a common cycle length. However, Lammer and Helbing[24] suggested that it is impossible to coordinate multiple intersections in such a system. Therefore, it is still an open question regarding whether signal coordination can be achieved without the requirement of a common cycle length.

Signal coordination models have some common Measures of Effectiveness (MOEs). Bandwidth maximization used to be a common objective function for signal coordination. It is the amount of time that a vehicle can travel through all intersections of the coordinated corridor without stopping. Bandwidth is related to the system capacity and throughput and is determined by the offsets. The literature on bandwidth optimization mostly relied on the graphical method in the early stage[24-28], which later focused on mixed integer linear programs (MILP) to maximize the sum of the bandwidths for the two directions of the coordinated corridor. Branch and bound algorithms were often used to solve the optimization problem. For example, Gartner *et al.*[29] expanded the previous signal coordination models by considering actual traffic volumes and flow capacity in the MILP formulation for bandwidth optimization. Their model is called MULTIBAND because they defined different bandwidths for each direction of the corridor, which were individually weighted based on their contributions to the objective value. PASSER is a software tool developed to maximize the bandwidth efficiency given the pre-calculated splits[30]. Other MOEs include delays, total travel times, and the number of stops when conducting offset optimization. Coogan *et al.*[31] optimized the offset of the coordinated traffic signals to reduce the average queue lengths at all intersections by assuming a fixed timing plan with a common cycle length. They derived a closed form analytical expression, which is a non-convex, quadratically constrained quadratic program (QCQP). The simulation results demonstrated a significant reduction in queue lengths. Hu and Liu[32] developed a data-driven arterial offset optimization model to minimize the total delay for the main coordinated direction,

which considered the stochastic nature of real-world traffic. They solved two problems in the proposed model: the early return to green problems for the coordinated phase and the uncertainty of the intersection queue size.

2.3 Traffic signal control with CVs

There have been various traffic signal control studies under the CV environment. Dual ring controllers have been applied to many of those studies. He *et al.*[33] developed the platoon-based arterial multi-modal signal control with online data (PAMSCOD) algorithm. Signal timing is updated every 30 seconds. A MINLP was solved to determine future optimal signal plan. Simulation results in VISSIM showed that delays were significantly reduced under both non-saturated and oversaturated traffic conditions compared to traditional state-of-the-practice coordinated actuated signal control. Lee and Park[34] developed a cumulative travel-time responsive (CTR) real-time intersection control algorithm in the CV environment. They examine the different penetration rates of CV and levels of congestion. Kalman filtering technique was utilized to estimate the cumulative travel time under various penetration rates. They suggested that 30% market rates of CV were needed to realize the benefits of the CTR algorithm. Feng *et al.*[35] presented a real time adaptive signal control algorithm using CV data. The algorithm incorporated a two-level optimization model with two objective functions: minimizing the vehicle delay and minimizing the queue length. Dynamic programming was applied to solve the discretized signal control problem. The cycle length is assumed to be variable. Beak *et al.*[36] extended the work of Feng *et al.*[35] in two ways. First, they imposed extra constraints to the upper level to ensure a fixed cycle length. Second, the revised intersection-level model (with a fixed cycle length) is integrated into a corridor-level model for signal coordination. They developed a two-level optimization method for adaptive coordination under the CV environment. At the intersection level, the optimal green time for each phase is determined from dynamic programing. At the corridor level, the offset is optimized to obtain minimum delay. They used a platoon model to estimate the platoon length and flow rate. A platoon dispersion model was applied to identify how a platoon disperses in the corridor over time. Simulation results show that the model can reduce average delay and number of stops for both coordinated phase and the entire network. Li and Ban[37] formulated the traffic signal optimization problem for a single intersection as a MINLP that was reformulated as a DP problem. They also developed a two-step method to make sure that the obtained optimal solution can lead to the fixed cycle length, which is often required for coordinating multiple signals on a traffic corridor or a network. Zhao *et al.*[38] proposed a signal timing optimization strategy to minimize the combined total energy consumption and traffic delay, considering the fuel consumption of individual vehicles. Vehicles' trajectories were predicted second by second using the Nagel-Schreckenber model. An iterative grid search algorithm was used to search for the optimized signal timing. There is also a large body of literature on traffic control with connected and automated vehicles (CAVs) for both intersection control and vehicle control; see Xu *et al.*[39]; Li *et al.*[40]; Li and Wang[41] and reviews in Li *et al.*[42].

There are also many studies that did not apply the dual-ring controllers in their CV-based signal optimization methods. These types of traffic signal designs are more flexible without considering the cycle length, the number of phases, phase transitions, or phase sequences. Although some of them still follow the phase-based signal design, they do not consider the cycle length or the offset in their designs. Priemer and Friedrich[43] proposed a decentralized adaptive traffic signal control using V2I communication data. Dynamic programming and complete enumeration were used to optimize the signal timing in order to reduce the total queue length within a forecast horizon of 20 seconds. Various penetration rates were tested in the simulation. Cai *et al.*[44] presented a traffic signal control algorithm using information collected from V2I. They constructed a state-space presentation of the control problem using the speed and position as state variables and applied dynamic programming and its derivative methods to optimize signal timing. Datesh *et al.*[16] applied the k-means clustering approach to improve the traffic signal efficacy. The algorithm is a platoon-based signal control method that categorizes the approaching vehicles into two groups, red or green. They demonstrated that the algorithm works properly under low penetration rates of CV. Goodall *et al*.[20] developed the predictive microscopic simulation algorithm (PMSA) to control traffic signal. The strategy can minimize total delays, or the combination of delays, stops, and decelerations over a 15-second time period by considering instantaneous vehicle data. The study showed that at low or mid-level traffic volume, their proposed algorithm outperformed state-of-the-practice coordinated-actuated timing plan, while the performance got worse during saturated and oversaturated conditions. However, the method ignored left-turn traffic and cannot be applied to real-world intersections. Li and Qiu[45] proposed an adaptive signal control approach based on CV to improve the intersection throughput. The approach incorporates a two-step centralized responsive control for vehicles in motion and stopped vehicles. The simulation results suggest that limited benefits are achieved when the traffic demand is high. Islam and Jadbabaie[46] developed a distributed coordinated methodology for signal timing optimization in CV networks. They reduced the complexity of a network level decision problem to a single intersection level problem by deciding the termination or continuation of green times. They evaluated the influence of demand levels and penetration rates of CV on their signal optimization algorithm in several case studies.

Among various types of methods, DP is one of the most commonly used techniques to solve the discretized signal control problems. It was first applied in Sen and Head[47] to optimize traffic signal timing. The idea was later applied in Chen *et al.*[48] and Feng *et al*.[35]. In particular, Feng *et al.*[35] proposed a bi-level formulation for optimizing signal timing of a single intersection: the upper level is to optimize for the barrier lengths and the lower level is to optimize for the phase times. However, all these studies assumed varying cycle lengths (and thus could not apply directly to deal with the fixed cycle length constraint). Signal timing plans with variable cycle lengths may not be readily applied to multiple intersections if signal coordination is needed. Beak *et al.*[36] extended Feng *et al.*[35] to impose the fixed cycle length, albeit with a bi-level formulation. First, they imposed extra constraints to the upper level to ensure a fixed cycle length. The revised intersection-level model (with a fixed cycle length) is then integrated into a corridor-level model

for signal coordination. In this project, a two-step method was developed to resolve the fixed cycle length issue at the intersection level. This avoids the use of the bi-level structure in Feng *et al.*[35] and Beak *et al.*[36], which can be more efficient in terms of computation.

For signal coordination under a CV environment, most of the existing traffic signal optimization/coordination methods applied a centralized scheme that various signal timing parameters (phase durations, cycle length, and offsets) are optimized together in one mathematical problem. This can lead to several problems. First, individual vehicle based signal control problems are often a NP hard problem[46]. Second, for a large traffic corridor or road network, the signal timing optimization and coordination problem is hard to solve and not applicable for real-time signal control. Third, some studies tried to decentralize the signal optimization problems by decomposing the entire problem into a few manageable sub-problems. However, they mostly assumed varying cycle lengths[34,43,46] and thus could not apply directly to traffic signal coordination.

This project aims to optimize the signal timing of a single intersection, along a corridor or for a network under the CV environment. At the intersection level, we assume a fixed cycle length for individual intersections so that the coordination can be achieved for multiple intersections. We first formulate the CV-based signal control problem as a MINLP. Due to the large dimension of the problem and the complexity of the nonlinear car-following model, solving the nonlinear program directly can be challenging. Secondly, we reformulate the problem as a DP model. We note that imposing the fixed cycle length constraint will invalidate the DP formulation. We then apply a two-step method to resolve this issue: end stage cost and branch and bound algorithm. Under corridor level optimization, the overall CV-based signal coordination problem is first formulated as a MINLP. The objective is to minimize the total weighted sum of fuel consumptions and travel times of all vehicles in the main street by calculating the optimal phase durations and offsets at the same time. Still the MINLP formulation has a large dimension. We then decompose the problem into a CV-based two-level traffic signal optimization and coordination scheme that contains an intersection level and a corridor level. In order to solve such a two-level model, we develop a prediction-based approach that collects the arrival vehicle information at the beginning of each cycle and calculates the optimal phase durations for each intersection using a DP method. During the calculation process, each intersection is aware of other intersections' decisions, traffic conditions, and the "temporary" optimal offsets. This ensures the traffic flows on the main street are coordinated at adjacent intersections. At the corridor level, the "temporary" optimal offsets are calculated iteratively in order to find the final optimal offsets until the total cost converges to the minimal value.

3. Traffic Signal Optimization with CVs

3.1 Methods overview

The objective of this project is to develop signal control strategies based on CV data, which has the potential to transform signal control at isolated intersections, along a corridor and for a network. This section provides an overview of the main methods that are developed in this project, including signal optimization methods for both a single intersection and multiple signals along a traffic corridor. More detailed discussions of each method are presented in subsequent sections.

Practical traffic signal control systems have different priorities. It is commonly agreed that the priorities of traffic signal control systems are (from high priorities to low priorities): safety, efficiency, and other objectives (such as fuel consumption and emissions). Safety can be ensured by well-established traffic control design methods that can separate conflicting traffic flows in time in order to reduce collisions. For example, a dual-ring controller can be applied to allocate the phase duration of each phase group, as shown in Figure 3. Due to its actuation features and safety concerns (minimal conflicts between different movements), dual ring phase controllers have become the dominant traffic signal type[50]. Dual ring controllers can also properly balance safety and efficiency of traffic signal control[10, 35]. This is important since the primary objective of traffic signal control is to ensure safety, i.e., to minimize movement conflicts[10], while mobility is also important as long as safety is ensured. Moreover, it can be easily set up and applied for signal coordination by setting a fixed cycle length constraint.

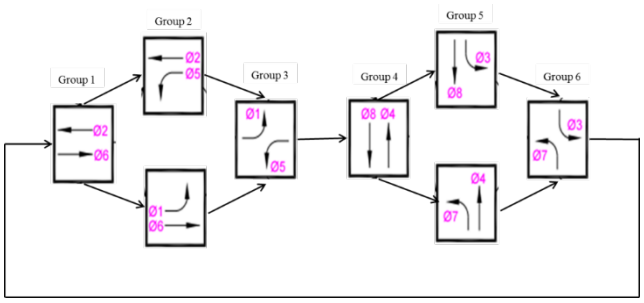


Figure 3: Traffic signal configuration [51]

Second, mobility and other objectives, such as fuel consumption and emissions, can be optimized by considering trajectories of all vehicles in a two-level traffic signal optimization method, with the intersection level to optimize the green time and corridor level to optimize the offset.

At the intersection level, the signal control problem can be first formulated as a MINLP by considering individual vehicles' trajectories (i.e., second-by-second vehicle locations and speeds) and their realistic driving/car-following behavior. The objective function is to minimize the weighted sum of total fuel consumption and travel time of all vehicles. Due to the large dimension of the problem and the complexity

of the nonlinear car-following model, solving the nonlinear program directly is challenging. We then reformulate the problem as a DP model by dividing the timing decisions into stages (one stage for a signal phase) and approximating the fuel consumption and travel time of a stage as functions of the state and decision variables. DP has advantages in formulation. If each stage corresponds to a phase, it can be formulated to calculate phase duration. Once the phase duration is zero, it means the phase is skipped in the optimal signal timing plan. Thus, DP is more flexible in phase sequence compared with other control methods.

Along a corridor or for a network, traffic coordination provides smooth progression to a platoon of vehicle traveling through multiple adjacent intersections with less delay and number of stops. In this project, the objective for corridor optimization/coordination is to produce optimal phase durations and offsets by minimizing the total fuel consumption and travel time of all vehicles traveling along the coordinated movements (i.e., on the main street). As shown in Figure 4, we can treat the bottom intersection as the reference signal and coordinate the other intersections based on the signal operations of the reference signal. Notice that the corridor shown in Figure 4 is an Eastbound-Westbound corridor. It is shown vertically in the figure in order to show the time-space diagram on the right. Usually the offset value is maintained for a period of time (e.g., 10 minutes) and may be changed based on the real traffic conditions.

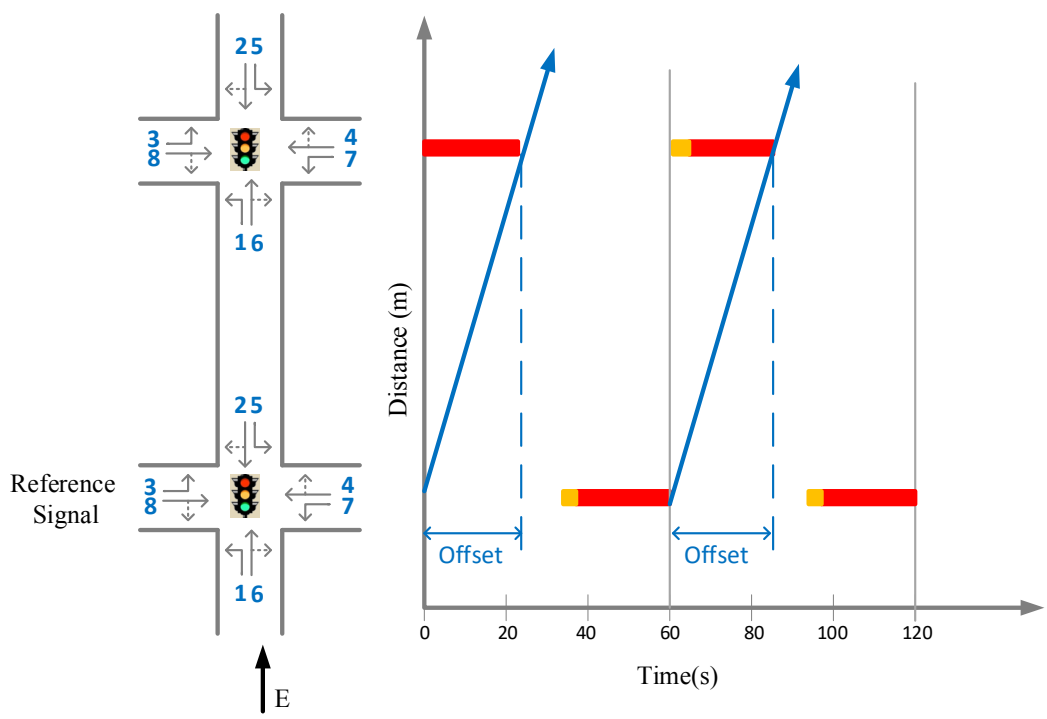


Figure 4: Coordination of multiple intersections

3.2 Intersection level optimization

This section presents the signal optimization method and numerical results for single intersection optimization. The idea of corridor level optimization will be present in the next section as future work. The signal control problem for single intersection with a fixed cycle length constraint is formulated as a MINLP. Here we adopt the dual-ring method for signal design as the signal configuration in a dual-ring diagram is shown in Figure 3. Without loss of generality, we assume the eastbound/ westbound (EB/WB) through movements (2 and 6 in Figure 3) are the major movements and thus cannot be skipped (i.e., for coordination purposes). Other phases may be skipped by setting the corresponding phase durations as zero. We also assume a cycle always starts with movements 2 and 6. Such a signal timing plan can be considered as 6 *groups* with a sequence of 8 *phases* in Figure 5. Note that phase 2 and 3 in group 2 cannot be realized at the same time, indicating that at least one of the two phases needs to be skipped. The same situation happens for phase 6 and 7 in group 5. In this paper, the continuous time is discretized into 1s intervals.

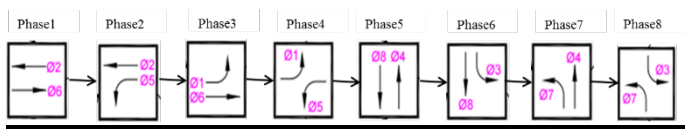


Figure 5: Traffic signal configuration [49]

3.2.1 Mixed-Integer Nonlinear Program

Parameters

C	Cycle length (s).
m_F	Monetary value of fuel ($/gal), e.g., $3/gal.
m_{TT}	Monetary value of travel time ($/s). e.g., $12/h ($0.005/s).
e	Idle fuel consumption rate (gal/h).
l_n	Length of vehicle n (m).
δ	Acceleration exponent in IDM. It usually set at 4.
H	Desired time headway (s), e.g., 1.5s.
a	Maximum acceleration rate ($\mathrm{m/s^2}$), e.g., 1 $\mathrm{m/s^2}$.
b	Maximum deceleration rate ($\mathrm{m/s^2}$), e.g., 3 $\mathrm{m/s^2}$.
s_{on}	Gap between vehicles in complete standstill traffic jams (m), e.g., 2m.
v_0	Vehicle desired speed (m/s).
g_k^{min}	Minimum effective green time of phase *k* (s).
g_k^{max}	Maximum effective green time of phase *k* (s).
k	Signal phase, *k* = 1, 2… 8.
$\bar{d}_n^0, \bar{d}_n^1$	Entrance location and exit location of the incoming approach of vehicle *n* (m).

$d_{signal,n}$	The location of the nearest front signal of vehicle n (m).
Variables	
$FC_{n,t}$	Fuel consumption for vehicle n at time *t* (gal/s).
$TT_{n,t}$	Travel time of vehicle n at time t (s).
$FC_{I,n}$	Fuel consumption for vehicle n at the idle status (gal/s).
$FC_{s,n,t}$	Fuel consumption of vehicle *n* at time *t* at the moving status (gal/meter).
g_k^i	Effective green time allocated to phase *k* of cycle *i* (s).
$v_{n,t}$	Speed of vehicle n at time t (m/s).
$d_{n,t}$	Location of vehicle n at time t (m).
$I_{n,t}$	Idle status indicator for vehicle *n* at time *t*.
$S_{k,t}$	Traffic signal status of phase group *k* at time *t*.
$\bar{k}$	Current phase index at time t. It represents the phase that is currently given the green light.
$Z_{n,t}$	Traffic signal status for vehicle *n* at time *t*.
$y_{n,t}$	Traffic signal indicator. It takes 1 if the preceding vehicle is traffic signal.
$s_{n,t}$	Vehicle gap (m).
$\Delta v_{n,t}$	Speed difference between vehicle n and $n-1$ at time t (m/s).
$a_{n,t}$	Acceleration rate for vehicle *n* at time *t* ($\mathrm{m/s^2}$).
$y_{t,1}, y_{t,2}, y_{t,3}, y_{t,4}$	Binary variables (auxiliary).

The objective of the CV-based signal optimization problem can be formulated as minimizing the weighted sum of total fuel consumption and travel time[46] of all vehicles approaching the intersection:

$$min\, F = \sum_{t=1}^{T} \sum_{n=1}^{N} \left(m_F FC_{n,t} + m_T T_{n,t} \right) \tag{1}$$

$FC_{n,t}$ and $T_{n,t}\mathrm{m_T T_n(t)}$ are the fuel consumption and travel time for vehicle n at time *t*. The corresponding parameters mF and mT are the "value of fuel" and "value of time" respectively. Eq. (1) indicates that the objective function here considers the travel time and energy consumption of individual vehicles. Eq. (2) calculates the fuel consumption of vehicle n at time t, which is determined by the vehicle status. If vehicle n is idling at time t, the indicator variable for idle status, $I_n(t)$ takes one and the fuel consumption model $FC_{I,n}$ is applied, as shown in Eq. (4). Otherwise, $FC_{s,n,t}$ will be used, as shown in Eq. (5), which calculates the fuel consumption of vehicle n at the moving status ($I_n(t) = 0$). Eq. (3b) reformulate (3a) using the "big *M*" method by establishing a relationship between speed $v_{n,t}$ and idle status indicator $I_{n,t}$. M here is a very large number. The model could be used to calculate fuel consumption for different vehicle types, including sedan, SUV, bus, electric vehicle (EV), and hybrid electric vehicle (HEV). Zhao *et al.*[46] provided the calibrated parameters in Eq. (4) and (5) for different vehicle types, as shown in Table 2.

$$FC_{n,t} = FC_{s,n,t} * v_{n,t} * \left(1 - I_{n,t}\right) + FC_{I,n} * I_{n,t} \tag{2}$$

$$I_{n,t} = \begin{cases} 1, & if\ v_{n,t} < 5, \quad idle\ at\ time\ t, \\ 0, & if\ v_{n,t} \geq 5, moving\ at\ time\ t \end{cases} \tag{3a}$$

$$\begin{cases} v_{n,t} - 5 < (1 - I_{n,t})M \\ v_{n,t} - 5 + I_{n,t}M \geq 0 \end{cases} \tag{3b}$$

$$FC_{I,n} = e \tag{4}$$

$$FC_{s,n,t} = \frac{a}{v_{n,t}} + b + cv_{n,t} + dv_{n,t}^2 \tag{5}$$

Vehicle Type	Parameters				
	a	*b*	*c*	*d*	*e*
1 EV	4.74e-2	2.66e-3	6.37e-5	1.49e-6	0
2 HEV (SOC0=0.7)	1.83e-1	3.67e-3	1.27e-4	2.39e-6	0
3 HEV (SOC0=0.6)	1.83e-1	3.67e-3	1.27e-4	2.39e-6	0
4 HEV (SOC0=0.5)	1.82e-1	1.51e-3	5.67e-4	-4.35e-6	0
5 Sedan	4.75e-1	-8.50e-3	5.41e-4	1.04e-7	0.211
6 SUV	7.44e-1	-1.23e-2	6.78e-4	5.29e-6	0.491
7 Bus	2.51e+0	3.03e-2	4.18e-3	-1.26e-5	1.184

Table 2: Parameter identification for fuel consumption models

As aforementioned, this study assumes the cycle length is fixed for the whole time span *T* (e.g., a few hours). The effective green time for each phase *k* of cycle *i*, g_k^i, must sum up to the (fixed) cycle length *C,* as shown in Eq. (6). There are eight phases in Figure 5, so *K* = 8. Eq. (7) indicates the bounds of the green time g_k^i. For phases that can be skipped, $g_k^{min} = 0$. Eq. (8-9) indicate phase 2 and 3 (and phase 6 and phase 7) cannot be realized for the same cycle i. At least one of the two variables, e.g., g_2^i (g_6^i) and g_3^i (g_7^i), need to be zero.

$$\sum_{k=1}^{K} g_k^i = C \qquad \forall\ i \in 1,2, \dots I \tag{6}$$

$$g_k^{min} \leq g_k^i \leq g_k^{max} \tag{7}$$

$$g_2^i * g_3^i = 0 \tag{8}$$

$$g_6^i * g_7^i = 0 \tag{9}$$

Each intersection contains multiple movements with each movement served by different phases k. Variable $S_{k,t}$ denotes the signal status at time t for phase k, as shown in Eq. (10a). It takes one if the signal status at the current time stamp is red and zero if it is green. The variable $\bar{k}$ is the current phase index at time t (i.e., 1, 2… 8). It represents the phase that is currently given the green light. Eq. (10b) reformulates (10a) using two binary variables $y_{t,1}$and $y_{t,2}$ based on the big *M* concept.

$$S_{k,t} = \begin{cases} 0, & if\ \sum_{k=1}^{\bar{k}-1} g_k^i \leq (t\ mod\ C) < \sum_{k=1}^{\bar{k}} g_k^i \\ 1, & otherwise \end{cases} \tag{10a}$$

$$\begin{cases} \sum_{k=1}^{\bar{k}} g_k^i - (t \bmod C) + y_{t,1}M > 0 \\ \sum_{k=1}^{\bar{k}} g_k^i - (t \bmod C) \leq (1 - y_{t,1})M \\ (t \bmod C) - \sum_{k=1}^{\bar{k}-1} g_k^i + y_{t,2}M \geq 0 \\ (t \bmod C) - \sum_{k=1}^{\bar{k}-1} g_k^i < (1 - y_{t,2})M \\ S_{k,t} = y_{t,1} + y_{t,2} \end{cases} \tag{10b}$$

Eq. (11) use indicator variables $y_{t,3}$and $y_{t,4}$ together to identify whether vehicle *n* is within the boundaries of the incoming approach: $\bar{d}_n^0$ and $\bar{d}_n^1$. Furthermore, the signal status $Z_{n,t}$ at time *t* for vehicle *n* could be determined as long as the incoming approach of vehicle *n* is identified, as shown in Eq. (12). Noted that signal status $Z_{n,t}$ and $S_{k,t}$ are different. Vehicles coming from different approaches may encounter different signal status (red or green).

$$\begin{cases} d_{n,t} - \bar{d}_n^1 \leq y_{t,3}M \\ d_{n,t} - \bar{d}_{n,}^1 + (1 - y_{t,3})M \geq 0 \\ d_{n,t} - \bar{d}_n^0 + y_{t,4}M \geq 0 \\ d_{n,t} - \bar{d}_n^0 \leq (1 - y_{t,4})M \end{cases} \tag{11}$$

$$Z_{n,t} = S_{k,t} - \frac{S_{k,t} + (y_{t,3} + y_{t,4}) - |S_{k,t} - (y_{t,3} + y_{t,4})|}{2} \tag{12}$$

The CV-based signal timing strategies in this paper require information on real-time vehicle trajectories. This project assumes a 100% penetration rate of connected vehicles. Vehicle trajectories can be transmitted when a vehicle enters the boundary of an intersection. Furthermore, to optimize signal timing for the current and future cycles, future vehicle trajectories are needed. For this, the Intelligent Driver Model (IDM)[52] is applied to simulate the vehicle trajectories. IDM is a car-following model that fits better with CV. We assume that there is only one lane per incoming approach, so there is no lane changing behavior involved. It is necessary to account for the signal status in the prediction of traffic flow propagation when applying IDM. For this, we model the red signal as a "standing vehicle" with speed equal to zero. It would disappear if the signal turns green. Eq. (13a) indicates whether the front object of vehicle *n* is a real vehicle or a standing vehicle (traffic signal) by comparing the relative location of the front vehicle $n-1$, vehicle n, and the nearest traffic signal in front of vehicle *n*. The binary variable $y_{n,t}$ takes one if the front "vehicle" is the traffic signal (could be red or green) at location $d_{signal,n}$ with speed zero. If $y_{n,t}$is zero, the front vehicle $n-1$ is a real vehicle with location $d_{n-1,t}$ and speed $v_{n-1,t}$. This helps update the vehicle trajectories in IDM as shown later. Eq. (13b) reformulate (13a) using the big *M* method and two binary variables $y_{n,t,1}$ and $y_{n,t,2}$.

$$y_{n,t} = \begin{cases} 1, if\ x_{n,t} < d_{signal,n} < x_{n-1,t} \\ 0, \qquad otherwise \end{cases} \tag{13a}$$

$$\begin{cases} d_{signal,n} - d_{n-1,t} < y_{n,t,1}M \\ d_{signal,n} - d_{n-1,t} + (1 - y_{n,t,1})M \geq 0 \\ d_{n,t} - d_{signal,n} < y_{n,t,2}M \\ d_{n,t} - d_{signal,n} + (1 - y_{n,t,2})M \geq 0 \\ y_{n,t} = 1 - (y_{n,t,1} + y_{n,t,2}) \end{cases} \tag{13b}$$

Eq. (*14-15*) identify the vehicle location and speed of the preceding "vehicle" $n-1$, which could be a real vehicle or the nearest front signal.

$$f_{n-1,t}^{d} = d_{n-1,t} * [1 - \frac{y_{n,t}+S_{n,t}-|y_{n,t}-S_{n,t}|}{2}] + d_{signal,n,t} * \frac{y_{n,t}+S_{n,t}-|y_{n,t}-S_{n,t}|}{2} \tag{14}$$

$$f_{n-1,t}^{v} = v_{n-1,t} * [1 - \frac{y_{n,t}+S_{n,t}-|y_{n,t}-S_{n,t}|}{2}] \tag{15}$$

Eq. (*16-19*) shows how IDM estimates the acceleration rate for vehicle *n* at each time interval, given the location and speed of vehicle $n-1$.

$$s_{n,t} = f_{n-1,t}^{d} - d_{n,t} - l_{n-1} \tag{16}$$

$$d_{n,t} = f_{n-1,t}^{v} - v_{n,t} \tag{17}$$

$$a_{n,t} = a[1 - \left(\frac{v_{n,t}}{v_0}\right)^{\delta} - \left(\frac{s^*(v_{n,t},d_{n,t})}{s_{n,t}}\right)^2] \tag{18}$$

$$s^*(v_{n,t}, d_{n,t}) = s_{on} + v_{n,t}H + \frac{v_{n,t}d_{n,t}}{2\sqrt{ab}} \tag{19}$$

Eq. (*20-21*) are applied to update the trajectories for vehicle *n* at next time interval $t+1$. More details of IDM can be found in Treiber *et al.* [52]. In this paper, the values of the parameters in IDM are chosen as *a* = $1m/s^2$, *b* =3 m/s^2, s_{on}= 2m, H = 1.5s, and δ = 4, according to Khondaker and Kattan [53].

$$v_{n,t+1} = max\,(0, v_{n,t} + a_{n,t}\,) \tag{20}$$

$$d_{n,t+1} = d_{n,t} + \frac{v_{n,t}+v_{n,t+1}}{2} \tag{21}$$

Eq. (1 – 21) is a MINLP for the CV-based signal control problem. It clearly shows that when individual vehicle status is considered for signal control, e.g., under the CV environment, the problem can be formulated as a very complex MINLP. This is mainly due to the different status of vehicles and signal phases, as well as the various if-then-else types of conditions (e.g., equations (3), (10), (13), and others) inherent to this coupled signal-vehicle optimization problem. In addition, since the variables of the model include the location and speed of each vehicle at each time interval, the dimension of the problem can be quite large. Furthermore, the IDM-based car-following model is also very complex. Thus, solving the model directly is quite challenging, and more tractable and efficient methods are needed. We next present one such method based on DP.

3.2.2 Dynamic programming formulation

DP provides a general framework to divide an optimization problem into multiple stages (under certain conditions), which could be solved sequentially one stage at a time. Here we divide the signal timing decisions into stages, one stage for a phase. We then approximate the total fuel consumption and travel time of a stage as functions of the state and decision variables. The notation is summarized as follows:

x_p	Decision variable, phase duration of stage *p* (s).
s_p	State variable, total time from beginning of the cycle to the end of stage *p* (s).
$X_p(s_p)$	The set of feasible control variable given stage variable s_p at stage *p* (s).
$V_p(s_p)$	Value function, the cumulative value of objective function from stage 1 up to stage *p* ($).
x^{min}	Minimum value of the decision variable (s).
$f_p(s_p, x_p)$	Total cost at stage *p*, given state variable s_p, and decision variable x_p ($).
N_p	Total number of vehicles in phase *p* (veh).
$FC_{n,t}(s_p, x$	Fuel consumption of the vehicle *n* at time t given stage variable s_p and decision variable x_p (gal/s).
$TT_{n,t}(s_p, x_{}$	Travel time of vehicle n at time t given stage variable s_p and decision variable x_p (s).
$FC_{s,n,t}(s_p,$	Fuel consumption of vehicle n at moving status at time t given stage variable s_p and decision variable x_p (gal/meter).
$FC_{I,n}(s_p, x$	Fuel consumption of vehicle n at idle status at time t given stage variable s_p and decision variable x_p(gal/s).
$A_p(s_{p-1}, s_{}$	The number of arriving vehicles in the time interval $[s_{p-1}, s_p]$.
$M_p(x_p)$	The maximum number of vehicle that can be discharged during phase duration x_p.
$v_{n,t}(s_p, x_{p}$	Approximated speed of vehicle *n* at time *t* given the stage variable s_p and decision variable x_p (m/s).
t_a	Arrival time at the predefined intersection boundary (distance *L* upstream of intersection) (s).
t_d	Time when vehicle joins the queue (started to slow down) (s).
t_0	Time when the vehicle fully stops (s).
t_{ac}	Time when vehicle starts to be discharged (s).
t_l	Time when the vehicle achieves the free flow speed v_0(s).
l_d	Distance upstream of end of queue or the stop line (if no queue) (m), e.g., 100m.
$V_p(s_p)$	Value function at phase p given state variable s_p.
σ	Tolerance of the fixed cycle length (s), e.g., 5s.

As shown in Figure 3 and Figure 5, there are eight stages in total. The state variable s_p is defined as the total number of time intervals from the beginning of the cycle to the end of stage *p*, while the decision variable x_p is the phase duration. Eq. (*22-23*) illustrate the relationship between state variable and decision variable; see Sen and Head[47] for more details.

$$s_p = s_{p-1} + h(x_p) \tag{22}$$

$$h(x_p) = \begin{cases} 0, \ if \ x_p = 0 \\ x_p + r, other \ wise \end{cases} \tag{23}$$

Given the state variable s_p, the feasible set of decision variables could be determined based on Eq. (*24*):

$$X_p(s_p) = \begin{cases} 0, & if s_p < x^{min} \\ \{0, x^{min}, x^{min} + 1, \dots, s_p\}, & otherwise \end{cases} \tag{24}$$

To formulate the DP, we first assign the initial value function $V_0 = 0$. The DP starts from stage (phase) *p* = 1, and proceed recursively to *p* = 2, 3 … 8. At each stage, the method calculates the optimal control decision variable $x_p^*(s_p)$ by minimizing the value function for each possible value of the state variable s_p in the forward recursion. Solving the DP is to find the shortest path in the graph. After the decision variable are estimated at all stages, the optimal decision of each stage can be retrieved.

However, in order to reformulate the signal control problem (*1–21*) as a DP, a critical condition is that the objective function in (*1*) can be expressed as the summation of the objective function of each stage. Furthermore, the stage-specific objective function (i.e., the sum of the vehicle fuel consumption and travel time of all vehicles in the stage) can be expressed as a function of the state and decision variables of that stage only[47]. This however is not true in general for most of the objectives we consider here, i.e., travel time or fuel consumption. It is especially so when we consider the data/information of individual vehicles (such as trajectories, speeds, delays, etc.). In the next subsection, we approximate the objective function of each stage so that it can be expressed as a function of the state and decision variables of the stage.

3.2.2.1 Objective function approximation

Eq. (*25*a) expresses the total fuel consumption and travel time of all the vehicles for phase *p* (i.e., it is from time s_{p-1} to s_p), where N_p is the total number of vehicles in phase *p*. In this paper, travel time of a vehicle is estimated by the summation of free flow travel time of the vehicle and the delay it encountered. As shown previously[47], the total delay (and thus travel time) of a stage in (25b) can be approximated as a function of the state and decision variables. We show in this subsection how the fuel consumption can be approximated as a function of the state and decision variables. As shown in (*5*) and rewritten in (25-), fuel consumption is a function of vehicle speed. Thus, vehicle speed should be approximated as a function of the state and decision variables.

$$Min \sum_{n=1}^{N_p} \sum_{s_{p-1}}^{s_p} f_p(s_p, x_p) \tag{25a}$$

$$f_p(s_p, x_p) \triangleq m_F FC_{n,t}(s_p, x_p) + m_t TT_{n,t}(s_p, x_p) \tag{25b}$$

$$FC_{n,t}(s_p, x_p) = FC_{s,n,t}(s_p, x_p) * v_{n,t}(s_p, x_p) * (1 - I_{n,t}) + FC_{I,n} * I_{n,t} \tag{26}$$

The speed of a vehicle is estimated based on the queue discharging process. Let $A_p(s_{p-1}, s_p)$ denote the number of arriving vehicles for phase *p*, i.e., during the time interval $[s_{p-1}, s_p]$. $M_p(x_p)$ denotes the maximum number of vehicles that can be discharged during the time interval of green time x_p. As shown in Sen and Head[47], A_p and M_p can be expressed as functions of the state and decision variables of stage *p* only. We next show how the *speed* of a vehicle can be approximated as those variables. There are four possible cases for the speed of a vehicle arriving in stage *p*, as shown in Figure 6.

(1) Vehicle arrives during green signal at stage p and can pass freely through intersection.

$$\bar{v} = v_o \tag{27}$$

(2) Vehicle arrives during green signal at stage p but a queue already exists. The queue includes vehicles that were not discharged in the previous stage p-1 and the newly arriving vehicles at current stage p before the current vehicle. Denote t_a the time when the current vehicle arrives at the predefined intersection boundary (distance l upstream of intersection), t_d the time when the vehicle joins the queue (started to slow down), t_0 the time when the vehicle fully stops, t_{ac} the time when the vehicle starts to be discharged, and t_l the time when the vehicle achieves the free flow speed v_0 again after being discharged. We assume the vehicle starts to decelerate with a constant rate at time t_d if queue exists. We can then approximate the average speed between t_d and t_0 as $\frac{v_0}{2}$. The same assumption applies to the acceleration process from t_{ac} to t_l. When the vehicle starts to decelerate at t_d, the distance between the vehicle and the stop line is denoted as l_d.

If the vehicle could pass the intersection within the current phase p (trajectory ① in Figure 6(b)), speed could be approximated using Eq. (28), otherwise (trajectory ②), vehicle has to wait until the next phase. In this case, we only apply the first three conditions in Eq. (28) since $t_1 \geq s_p$. We could consider ② as a special case of ①.

$$v_{n,t}(s_p, x_p) = \begin{cases} v_0, & if\ t_a \leq t \leq t_d \\ \frac{v_0}{2}, & if\ t_d < t \leq t_0 \\ 0, & if\ t_0 < t \leq t_{ac} \\ \frac{v_0}{2}, & if\ t_{ac} < t \leq t_l \\ v_0, & if\ t_l < t \leq s_p \end{cases} \tag{28}$$

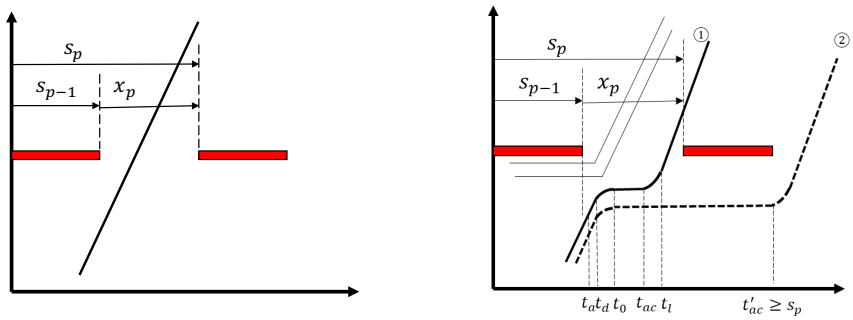
(3) Vehicle arrives during red signal at stage p. Figure 6 (c) and (d) are differentiated by whether queue exists at the end of stage p. If there is no queue, vehicle n will start to leave at $t_1(t_1 = s_p)$, otherwise it will have to wait for the queue to dissipate ($t_1 \geq s_p$). Here we only care about the vehicle status from time s_{p-1} to s_p. The speed of vehicle n arriving during red can be summarized as:

$$v_{n,t}(s_p, x_p) = \begin{cases} v_0, & if\ t_a \leq t \leq t_d \\ \frac{v_0}{2}, & if\ t_d < t \leq t_0 \\ 0, & if\ t_0 < t \leq s_p \end{cases} \tag{29}$$

After approximating the vehicle speed $\boldsymbol{v_{n,t}(s_p, x_p)}$ in the objective function for stage p based on the above four cases, the objective function can be approximated as a function of $\boldsymbol{s_p}$ and $\boldsymbol{x_p}$ only, as shown below:

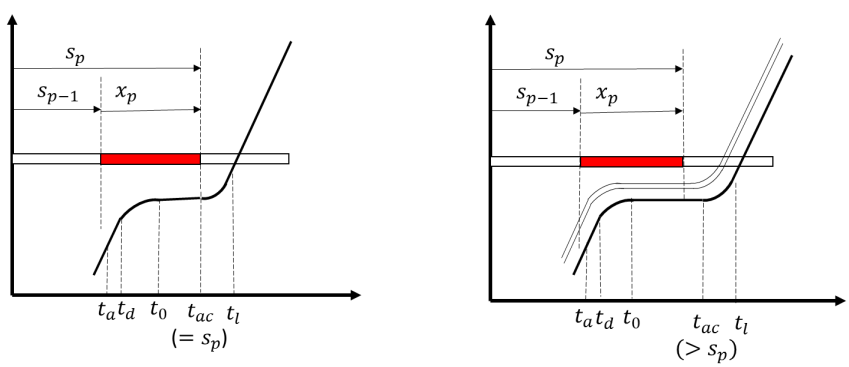
$$V_p(s_p) = min\{f_p(s_p, x_p) + V_{p-1}(s_{p-1}) | x_p \in X_p(s_p), p \in P\} \tag{30}$$

This means that Eq. (25), and the objective function of the MINLP in Section II as well, can be expressed as a function of $\boldsymbol{s_p}$ and $\boldsymbol{x_p}$ only. Thus the problem can be reformulated as a DP. Detailed proof is straightforward and omitted here. Furthermore, after obtaining the optimal decision variable $\boldsymbol{x_p^*}$ at stage p, IDM is applied to update the trajectories of all the vehicles from $\boldsymbol{s_{p-1}}$ to $\boldsymbol{s_p = s_{p-1} + x_p^*}$.



(a) Vehicle arrives during green signal (no queue)

(b) Vehicle arrives during green signal and queue exists



(c) Vehicle arrives during red signal (no queue)

(d) Vehicle arrives during red signal and queue exists

Figure 6: Four cases for approximating the vehicle average speed

3.2.2.2 End stage cost

The DP formulation so far does not impose any constraint on the cycle length. As shown in the next section, imposing such a constraint will invalidate the DP formulation. We propose two steps to address this issue: adding the end-stage cost to the DP formulation and a branch-and-bound method to refine the

solution. First, in the last stage P, we modify the value function by adding the end stage cost f_e if the difference between the estimated cycle length C_e by DP and the predefined cycle length C is large. If σ denotes the tolerance, e.g., 5 seconds, and w denotes the weight, the revised value function for the last stage P can be expressed as:

$$V_p(s_p) = min\{f_p(s_p, x_p) + V_{p-1}(s_{p-1}) + f_e | x_p \in X_p(s_p), p = 8\} \tag{31}$$

The end-stage cost f_e can be expressed as in Eq. (*32*) while the estimated cycle length is the summation of the green time of all phases:

$$f_e = \begin{cases} 0, & if\ |C_e - C| \leq \sigma \\ w(C_e - C)^2, & otherwise \end{cases} \tag{32}$$

$$C_e = \sum_{p=1}^{P} x_p \tag{33}$$

The end-stage cost in Eq. (*32*) with a proper weight works as a penalty function, ensuring that the obtained solution from the DP can result in a cycle length Ce close enough to the predefined cycle length C. To obtain a solution that can lead to C exactly, the second step applies a branch and bound method to refine the DP solution.

3.2.2.3 Branch and bound algorithm

To produce a signal timing solution that leads to exactly the predefined cycle length *C*, we need to add the following constraint to the decision variables in the DP formulation.

$$\sum_{p=1}^{8} x_p = C \tag{34}$$

This constraint, however, will invalidate the DP model as decisions cannot be made stage by stage when Eq. (*34*) is considered. In other words, DP cannot guarantee that the optimal phase duration sums up to a fixed cycle length. A branch and bound method is applied here to resolve the issue, which was used in the past to solve the Resources Constrained Shortest Path (RCSP) problem[54]. The method creates a tree by selecting one variable each time from an initial solution. Here the initial solution is produced by solving the DP formulation with the end-stage cost, as discussed above. The maximum level of the tree is the number of stages because the variable in a given set can only be used for branching once. Moreover, all branches at a given level of the tree have to be computed and analyzed before advancing to the next level. The numerical example and more explanations of the branch and bound method will be provided in the numerical results section. The branch and bound method can be summarized as below:

(1) DP with the end-stage cost is solved first to produce an initial solution.

(2) Define the error gain for stage/phase *p*:

$$ER_p(s_p, x_p) = f_p(s_p, x_p) \tag{35}$$

The error gain is the objective function in DP (total fuel consumption and travel time) for all vehicles traveling in stage p with stage variable s_p and decision variable x_p.

(3) If the estimated cycle length happens to be exactly the predefined cycle length C, the DP solution is an optimal solution. The algorithm stops.

(4) If the cycle length from the DP solution is larger than the predefined fixed cycle length (decision variable should be decreased), the selected phase for branching at each level is the one that has the minimum error gain. There could be multiple numbers of branching depending on the difference between the predefined cycle length and the cycle length from the DP solution. If the produced cycle length is less than the fixed cycle length (decision variable should be increased), the selected phase for branching would be the maximum error gain.

(5) The algorithm stops when the results of all branching (i.e., leaves) are feasible. The optimal solution is selected from the feasible solutions that satisfies the cycle length constraint while producing the minimum objective value.

3.3 Corridor level optimization

3.3.1 Formulating signal coordination as a mixed-integer nonlinear program

As shown in section 143.2.1, the signal control problem for a single intersection can be formulated as a MINLP. It can produce the optimal phase durations that satisfy fixed cycle length constraint. The previous model can be extended to a corridor level by introducing the offset variable O_j for intersection *j*, when considering the coordination of multiple intersections on the corridor. The objective is to minimize the weighted sum of fuel consumption and travel times for all vehicles (the total number is *N*) on the main street for the total time span *T*, as shown in Eq. (36). $FC_{n,t}$ and $TT_{n,t}$ are the fuel consumption and travel time for vehicle n at time *t*. The corresponding parameters m_F and m_{TT} are the "value of fuel" and "value of time" respectively[49]. The fuel consumption $FC_{n,t}$ can be estimated using Eq. (2-5).

$$min\, F = \sum_{t=1}^{T} \sum_{n=1}^{N} \left(m_F FC_{n,t} + m_{TT} TT_{n,t}\right) \tag{36}$$

In order to coordinate multiple intersections, we need to establish connections between coordinated phases for different traffic signals, e.g., start of green time of phase 1 (movement 2 and 6 in Figure 5) for each intersection. These connections are represented by two types of time stamps: global time *t* and local time t_j'. Global time index t refers to the master clock to which each signal is referenced during coordinated operations. It starts from zero up to the total time span *T*. Local time t_j' refers to the time

stamp for a local intersection j starting from zero up to cycle length C. The relationship between t and t_j' can be present by Eq. (37):

$$t_j' = mod(t - O_j,\ C), 0 \le t_j' \le C \tag{37}$$

where *mod* refer to modulo operation. Offset O_j is the time difference of the starting times of the coordinated phase between the reference signal and signal j. The boundary condition is shown in Eq. (38), where O^{max} denotes the parameter of maximum offset.

$$0 \le O_j \le O^{max} \tag{38}$$

The fixed cycle length constraint is necessary in order to conduct signal coordination. It requires that the effective green time for each phase *k* of cycle *i* at intersection *j* sums up to the fixed cycle length *C*, as shown in Eq. (39). Eq. (40) shows the boundary of green time at phase k of cycle i at intersection j. Eq. (41-42) indicates that phase 2 and 3 (phase 6 and 7) cannot be realized in the same cycle at intersection j. In other words, one of the two variables, e.g., $g_{j,2}^i$ ($g_{j,6}^i$) and $g_{j,3}^i$ ($g_{j,7}^i$), needs to be zero.

$$\sum_{k=1}^{K} g_{j,k}^i = C \qquad \forall\, i_j \in 1,2,\dots I \tag{39}$$

$$g_{j,k}^{min} \le g_{j,k}^i \le g_{j,k}^{max} \tag{40}$$

$$g_{j,2}^i * g_{j,3}^i = 0 \tag{41}$$

$$g_{j,6}^i * g_{j,7}^i = 0 \tag{42}$$

In order to estimate the fuel consumption and travel time of all vehicles in the network, we need to know the signal status at each intersection at each time stamp because it will influence vehicle trajectories. Variable $S_{j,k,t}$ denotes the signal status at time *t* for phase k at intersection j, as shown in Eq. (43a). It takes one if the signal status at time t_j' at intersection j is red and zero if it is green. The variable $\bar{k}$ is the current phase index at time t_j' (i.e., 1, 2… 8). It represents the phase that is currently given the green light. Eq. (43b) reformulates (43a) using two binary variables $y_{t,1}$and $y_{t,2}$ based on the big *M* concept **Error! Reference source not found.**.

$$S_{j,k,t} = \begin{cases} 0, & if\ \sum_{k=1}^{\bar{k}-1} g_{j,k}^i \le t_j' < \sum_{k=1}^{\bar{k}} g_{j,k}^i \\ 1, & otherwise \end{cases} \tag{43a}$$

$$\begin{cases} \sum_{k=1}^{\bar{k}} g_{j,k}^i - t_j' + y_{t,1}M > 0 \\ \sum_{k=1}^{\bar{k}} g_{j,k}^i - t_j' \le (1 - y_{t,1})M \\ t_j' - \sum_{k=1}^{\bar{k}-1} g_{j,k}^i + y_{t,2}M \ge 0 \\ t_j' - \sum_{k=1}^{\bar{k}-1} g_{j,k}^i < (1 - y_{t,2})M \\ S_{j,k,t} = y_{t,1} + y_{t,2} \end{cases} \tag{43b}$$

After identifying the signal status at each intersection at each time stamp, we need to find the nearest front signal of vehicle n based on the relative positions of signals and itself. Eq. (44) uses indicator variables $y_{t,3,j}$and $y_{t,4,j}$ together to identify whether vehicle n is within the boundaries of the intersection *j:* $\bar{d}_{j,n}^0$ and $\bar{d}_{j,n}^1$. It helps identify the nearest traffic signal in front of vehicle *n* at time *t*. Furthermore, the

signal status $Z_{n,t}$ at time *t* for vehicle *n* could be determined as long as the nearest traffic signal $\bar{j}$ in front of vehicle n is identified, as shown in Eq. (46).

$$\begin{cases} d_{n,t} - \bar{d}_{j,n}^{1} \leq y_{t,3,j}M \\ d_{n,t} - \bar{d}_{j,n}^{1} + (1 - y_{t,3,j})M \geq 0 \\ d_{n,t} - \bar{d}_{j,n}^{0} + y_{t,4,j}M \geq 0 \\ d_{n,t} - \bar{d}_{j,n}^{0} \leq (1 - y_{t,4,j})M \end{cases} \tag{44}$$

$$\bar{j} = \sum_{j=1}^{J} j(1 - (y_{t,3,j} + y_{t,4,j})) \tag{45}$$

$$Z_{n,t} = S_{\bar{j},k,t} - \frac{S_{\bar{j},k,t} + (y_{t,3,\bar{j}} + y_{t,4,\bar{j}}) - |S_{\bar{j},k,t} - (y_{t,3,\bar{j}} + y_{t,4,})|}{2} \tag{46}$$

As long as we identify the signal status of the nearest signal in front of vehicle *n*, its trajectories can be estimated and predicted based on the intelligent driving model, i.e., IDM, which follows the same principals in the single intersection model in Eq. (13a-21). Eq. (47-55) are identical to Eq. (13a-21). They are listed in this section in order to show the complete MINLP.

$$y_{n,t} = \begin{cases} 1, if\ x_{n,t} < d_{signal,n} < x_{n-1,t} \\ 0, \qquad otherwise \end{cases} \tag{47a}$$

$$\begin{cases} d_{signal,n} - d_{n-1,t} < y_{n,t,1}M \\ d_{signal,n} - d_{n-1,t} + (1 - y_{n,t,1})M \geq 0 \\ d_{n,t} - d_{signal,n} < y_{n,t,2}M \\ d_{n,t} - d_{signal,n} + (1 - y_{n,t,2})M \geq 0 \\ y_{n,t} = 1 - (y_{n,t,1} + y_{n,t,2}) \end{cases} \tag{48b}$$

$$f_{n-1,t}^{d} = d_{n-1,t} * [1 - \frac{y_{n,t} + Z_{n,t} - |y_{n,t} - Z_{n,t}|}{2}] + d_{signal,n,t} * \frac{y_{n,t} + Z_{n,t} - |y_{n,t} - Z_{n,t}|}{2} \tag{48}$$

$$f_{n-1,t}^{v} = v_{n-1,t} * [1 - \frac{y_{n,t} + Z_{n,t} - |y_{n,t} - Z_{n,t}|}{2}] \tag{49}$$

$$s_{n,t} = f_{n-1,t}^{d} - d_{n,t} - l_{n-1} \tag{50}$$

$$d_{n,t} = f_{n-1,t}^{v} - v_{n,t} \tag{51}$$

$$a_{n,t} = a[1 - \left(\frac{v_{n,t}}{v_0}\right)^{\delta} - \left(\frac{s^*(v_{n,t}, d_{n,t})}{s_{n,t}}\right)^2] \tag{52}$$

$$s^*(v_{n,t}, d_{n,t}) = s_{on} + v_{n,t}H + \frac{v_{n,t}d_{n,t}}{2\sqrt{ab}} \tag{53}$$

$$v_{n,t+1} = max\ (0, v_{n,t} + a_{n,t}\ ) \tag{54}$$

$$d_{n,t+1} = d_{n,t} + \frac{v_{n,t} + v_{n,t+1}}{2} \tag{55}$$

Eq. (36-55) is a MINLP for the CV-based signal optimization and coordination for multiple intersections on a traffic corridor. When trajectories of individual vehicles are considered, it clearly shows that the problem can be formulated as a MINLP. Compared to the single intersection model in section 3.2.1, model (36-55) is even more complex due to the relationships between different statuses of vehicles and multiple traffic signals, various if-then-else conditions, and the large dimension of the problem (considering the location and speed of each CV). As a result, for a large corridor or a road network, the MINLP formulation is hard to solve and not applicable for real-time signal control. When solving the MINLP directly, a local optimum

is often produced instead of the global optimal solution. In the next section, the traffic signal optimization and coordination problem (36-55) is decomposed into a decentralized two-level model that is able to solve the problem more efficiently.

3.3.2 Reformulating signal coordination as a two-level model

Due to the complexity of the MINLP formulation (36-55), we reformulate the MINLP as a decentralized two-level problem. In this two-level model, instead of solving the phase durations and offsets optimization for multiple intersections as one mathematical program, we decompose the overall problem into two levels: an intersection level to optimize the phase durations for each single intersection and a corridor level to optimize the offsets. As a result, the complexity of the original mathematical problem is significantly reduced.

Figure 7 shows the overall framework of the two-level model. It is assumed that all intersections and vehicles are connected and they can send messages to each other. It further assumes some initial offsets and phase durations are available, e.g., those generated by SNYCHRO, a widely used traffic signal design and optimization software tool. The arrival vehicles to each intersection can then be estimated and predicted using IDM. Such information is critical input to the DP method as detailed in Section 3.2.2 to optimize the timing plans for single intersections. In particular, the DP method is applied to compute the optimal phase durations in every cycle of each intersection by minimizing the total weighted sum of fuel consumption and travel time of all CVs within the boundary of the intersection, e.g., 400m upstream and 400m downstream of the intersection. Section 3.2.2 showed that DP with a two-step method (end stage cost and branch and bound algorithm) is able to generate the phase durations that guarantee the given fixed cycle length for a single intersection. Figure 8 illustrates the DP calculation process, which can be equivalently represented as an acyclic graph. Solving the DP is to find the shortest path in the graph. The state variable s_p here is defined as the total number of time intervals from the beginning of the cycle to the end of stage *p*, while the decision variable x_p is the phase duration. The variable offset provides the connection between the intersection and corridor level optimization. In order to consider the influence of offsets on DP implementation for an intersection, we follow the rules in Eq. (37) that DP for intersection *j* will be implemented using its local time t_j'. This essentially reduces the model (36-55) for the entire corridor to each individual intersection.

After the optimal signal plans for all intersections on a corridor are generated for a few cycles, such information is transferred to a central controller where offset optimization is conducted at the corridor level. Usually the offset values are maintained for a period of time and not updated in every cycle. Here the offsets are assumed to be updated in every *Nc* cycles in Figure 7, e.g., in every 10 cycles. We further reformulate the offset optimization problems as a MINLP in this section, but with much fewer variables since the phase durations for each intersection are given from solving the intersection-level model (using

the DP method in Section 3.2.2) above. The objective of the corridor-level optimization is to minimize the total weighted sum of fuel consumption and travel time of all CVs on the main street of the corridor:

$$min\, F = \sum_{t=1}^{T} \sum_{n=1}^{\bar{N}} \left(m_F FC_{n,t} + m_{TT} TT_{n,t} \right) \tag{56}$$

where $\bar{N}$ is the total number of vehicles on the main street. The constraints on the offsets are identical to those in Eq. (37-38). Since the optimal green time for phase *k* of cycle i at intersection *j* is generated from the intersection-level model, the variable $g_{j,k}^{i}$ in the original MINLP model (36-55) becomes a parameter $\bar{g}_{j,k}^{i}$ in the corridor-level model. The signal status at intersection *j* for phase *k* at time *t* can be estimated from Eq. (57a). Eq. (57b) reformulate Eq. (57a) using big *M* constraint.

$$S_{j,k,t} = \begin{cases} 0, & if\ \sum_{k=1}^{\bar{k}-1} \bar{g}_{j,k}^{i} \le t_j' < \sum_{k=1}^{\bar{k}} \bar{g}_{j,k}^{i} \\ 1, & otherwise \end{cases} \tag{57a}$$

$$\begin{cases} \sum_{k=1}^{\bar{k}} \bar{g}_{j,k}^{i} - t_j' + y_{t,1} M > 0 \\ \sum_{k=1}^{\bar{k}} \bar{g}_{j,k}^{i} - t_j' \le (1 - y_{t,1}) M \\ t_j' - \sum_{k=1}^{\bar{k}-1} \bar{g}_{j,k}^{i} + y_{t,2} M \ge 0 \\ t_j' - \sum_{k=1}^{\bar{k}-1} \bar{g}_{j,k}^{i} < (1 - y_{t,2}) M \\ S_{j,k,t} = y_{t,1} + y_{t,2} \end{cases} \tag{57b}$$

After the signal status is identified, vehicle trajectories can be estimated from IDM using Eq. (44-55). This constitutes the corridor-level model that is also a MINLP but much simpler than the original MINLP (36-55). The corridor-level model has much fewer variables (only the offsets) and can be solved by the NOMAD solver in Matlab. NOMAD uses a Mesh Adaptive Direct Search algorithm to solve non-differentiable and global nonlinear programs. It computes/updates vehicle speed and location at each time step using IDM. As shown in Figure 7, in order to find the optimal offsets, "temporary" optimal offsets generated from the corridor-level MINLP are calculated iteratively until the total cost converges. The convergence criterial is defined as the difference between the total costs estimated from two consecutive iterations, i.e., when ΔTC is less than a small tolerance, e.g., 10^{-5}.

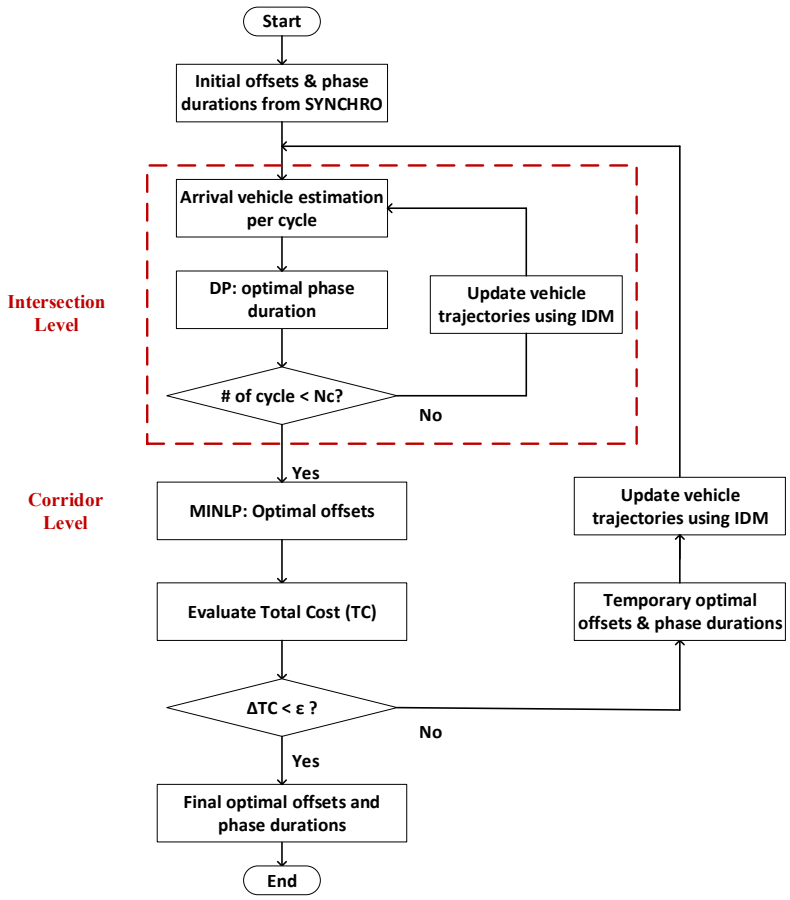


Figure 7: Solution technique of the two-level traffic signal optimization model

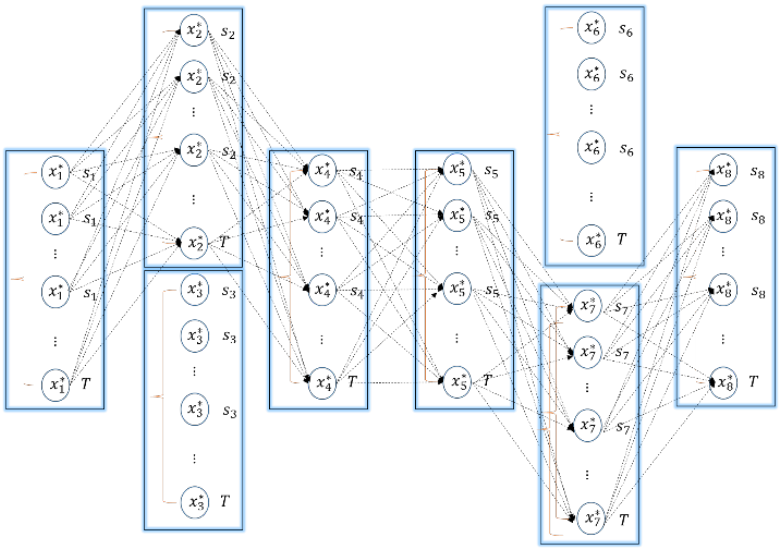


Figure 8: Acyclic graph of A DP Formulation

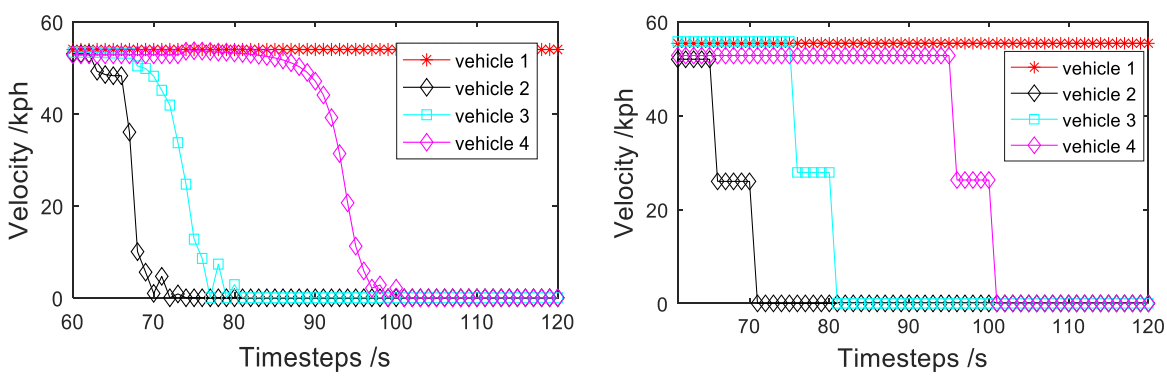
4. Results

4.1 Single intersection

The proposed CV-based signal timing optimization model and DP method are tested using data generated via traffic simulation in VISSIM. The testing network contains a single intersection with a boundary of 300 meters upstream and downstream of the intersection to mimic the communication range of V2I. The σ parameter in Eq. (32) is set as 5. After comparing the approximated vehicle speed using our approximation method and the speed generated from VISSIM, we evaluate the proposed signal timing optimization method in three steps. First, we estimate the optimal cycle length using SYNCHRO, a widely used traffic signal design and optimization software tool, for different traffic demand cases. Next, for a given case, we apply different methods (see Table 3 and Table 4) to optimize the signal timing plans (phases and green splits), one for each method. We then evaluate the performance of each signal timing plan (i.e., each method) by applying the plan to the intersection and using IDM to generate vehicle trajectories, based on which the total cost is calculated. We also illustrate the procedure of the branch and bound algorithm and test the impact of the tolerance in the end-stage cost Eq. (*32*) on the performance of the proposed model.

4.1.1 Speed approximation

In the proposed method, we approximate the vehicle speed as a function of the state variables and decision variables, as shown in Section III. Here we compare the vehicle speed between our method (Eq. (*28-30*)) and IDM simulation for the major streets (link 2 and link 4) in Figure 9. The platoon contains four vehicles. It is observed that the approximate speeds have a similar trend but not as smooth a speed profile as in IDM.



(a) IDM speed

(b) Approximated speed

Figure 9: Speed Comparisons

4.1.2 Signal timing optimization

Different combinations of traffic demands and vehicle types are tested in order to evaluate the proposed signal optimization method. Vehicle arrivals at the boundaries of the intersection are generated from VISSIM. For each direction, 80% of the vehicles use through lanes and the others use left lanes. Six cases are tested to identify the influence of traffic demands and vehicle types on the model performance. In Case I – III, vehicle demand is set to be 250 vph, 500 vph, and 800 vph, respectively. All vehicles are sedans (Type 5 in Table 2). In Case IV - VI, traffic demands are identical as in Case I - III, but the vehicle types are assigned differently. In the N-S directions, vehicles are assigned as EV, i.e., Vehicle type 1, while in the W-E directions, vehicles are assigned as buses (Vehicle type 7).

We test all models for 10 cycles. There are vehicles randomly generated to enter the network during the first 8 cycles, while during the last 2 cycles, there is no traffic demand. This guarantees the network is cleared by the end of the simulation. We test three signal control methods, as shown in Table 3 and Table 4. The first method is the actuated signal timing plan produced by SYNCHRO. The timing plan from SYNCHRO is then applied in IDM to update vehicle trajectories and estimate the objective function value, i.e., the total cost of fuel consumption and travel time using Eq. (1-5). The same procedure also applies to the other two models: the NOMAD solver in MATLAB and the DP method proposed here. The solver "NOMAD" uses a Mesh Adaptive Direct Search algorithm to solve non-differentiable and global nonlinear programs. It can solve non-convex MINLPs, but may not produce the global optimal solution. Since the starting point will directly influence the optimization results in NOMAD, we set the solution from SYNCHRO and DP respectively as the starting point in NOMAD to further reduce the cost. Dimensionality is another key factor affecting the performance of NOMAD. We test the model by updating the signal plan at various intervals (e.g., in every 1, 2, 5 or 10 cycles). The number of variables in NOMAD increases as the update frequency increases. For example, if we update the signal plan every cycle, there will be 80 variables in the total time span (10 cycles). The third model is the DP with the proposed end stage cost and the branch and bound method. DP updates the signal plan in every cycle. The results from DP without fixed cycle length constraints are also shown in parentheses in Tables 3 and 4.

Table 3 and Table 4 summarize the results, in terms of the objective values, by different models for the six cases respectively, considering the influence of demand levels only (Table 2) and the combined demand levels and vehicle types (Table 3). The cycle lengths for different demand levels are determined by SYNCHRO: they are 60s for low traffic demand (250 vph), 65s for medium traffic demand (500 vph), and 85s for high demand (800vph), which is also the maximum value for the state variables in DP. Table 3 shows the cost from different models under various demand levels. NOMAD produces different solutions for different signal plan updating intervals and starting points (from SYNCHRO in first four rows and from DP in the last row). The best solutions from NOMAD are highlighted for each Case. Table 3 shows that if the signal plan is updated every cycle or every two cycles, the network performance is not improved, as the total costs stay the same as the initial evaluation from the initial guess (SYNCHRO plan). However, a

better solution may be obtained as the updating frequency decreases to every 5 or 10 cycles. This indicates that NOMAD has difficulties finding optimal solutions when the number of variables is relatively large. Table 3 also shows that different starting points in NOMAD can affect the optimal solutions. When compared with different models, DP outperforms SYNCHRO and achieves the same solution as NOMAD under low and medium demands in Case I and Case II. In Case III, as the demand increases, DP results are still better than SYNCHRO but slightly worse than NOMAD. Table 4 shows the costs from different models under various demand levels and vehicle types. By comparing Case IV to Case I, we can see that the performance improvements of DP and NOMAD over SYNCHRO are more dramatic, which is also shown in Figure 10. Case IV incorporates buses in W-E direction, which produces much more fuel consumption than sedans in Case I. The cost generated from DP and NOMAD were both lower than SYNCHRO because SYNCHRO does not consider the influence of vehicle types on fuel consumption when optimizing the signal timing. As the demand increases while still maintaining different vehicle types on NS and WE directions, the performance of DP is still better than SYNCHRO but not as good as NOMAD in medium (500vph) and high (800 vph) demand levels. Since NOMAD uses the optimal phase plan from SYNCHRO and DP as the starting point, it makes sense that the best NOMAD results are always better than or equal to SYNCHRO and DP for all cases. It is also observed that the results from DP without the fixed C constraint always have lower objectives compared with the one with the fixed C constraint. This indicates that enforcing the fixed cycle length, i.e., for signal coordination purposes (and thus to the benefit of the entire system), is likely to worsen the performance of individual intersections.

Model	Case I	Case II	Case III	# of variables
	250 vph, sedan	500 vph, sedan	800 vph, sedan	
1. SYNCHRO	49.64	162.39	368.99	/
2. NOMAD update every cycle	49.64	162.39	368.99	80
NOMAD update every 2 cycles	49.64	162.39	368.99	40
NOMAD update every 5 cycles	49.64	153.25	362.5	16
NOMAD update every 10 cycle	48.12	143.52	358.05	8
NOMAD update every 10 cycles & initial points from DP solution)	47.74	140.65	359.96	8
3. DP with fixed C constraint	47.74	140.65	360.73	/
(DP without fixed C constraint)	46.97	138.55	354.74	

Table 3: Cost of Different Models under Various Demand Levels

Model	Case IV 250 vph; NS: Evs; WE: Bus	Case V 500 vph; NS: Evs; WE: Bus	Case VI 800 vph; NS: Evs; WE: Bus	# of variables
1. SYNCHRO	68.42	209.54	453.81	/
2. NOMAD update every cycle	68.42	209.54	453.81	80
NOMAD update every 2 cycles	68.42	209.54	453.81	40
NOMAD update every 5 cycles	68.42	168.29	453.81	16
NOMAD update every 10 cycle	65.78	180.67	434.37	8
NOMAD update every 10 cycles & initial points from DP solution)	63.37	169.54	436.65	8
3. DP with fixed C constraint	63.37	172.43	436.65	/
(DP without fixed C constraint)	61.25	163.54	433.36	

Table 4: Cost of Different Models under Various Demand Levels and Vehicle Types

Figure 10 shows the performance improvements of NOMAD, DP with fixed cycle length and DP without fixed cycle length from SYNCHRO. As shown by the dashed line, the model improvements of low and high demand levels are not as significant as the middle demand levels. This may be because under unsaturated but relatively heavy traffic conditions, there are more opportunities to optimize the splits and reduce the total cost of fuel consumption and travel time. Such opportunities tend to diminish when traffic is very light (all methods can work well) or very heavy (no method can work well). Furthermore, the performance improvements are more obvious if considering different vehicle types, as shown in Case IV – VI in Figure 10.

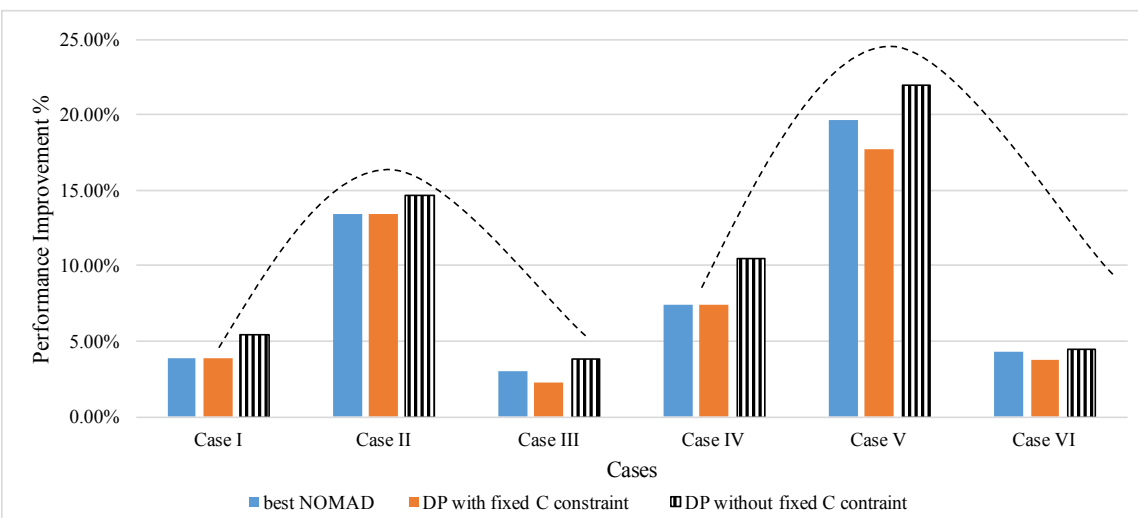


Figure 10: Improvement of Model Performance over SYNCHRO Results

Figure 11 shows the cost of fuel consumption and travel time separately for the four methods and six cases. For all cases, the cost of travel time is much larger than the cost of fuel consumption. Comparing Cases IV, V, VI to Cases I, II, III, it is observed that the influence of vehicle

types is more significant on the cost of fuel consumption than travel time. As shown in Figure 11, considering the same level of travel demand (e.g., Cases I and IV), the cost of fuel consumption is larger for the cases considering different vehicle types while the costs of travel time are similar.

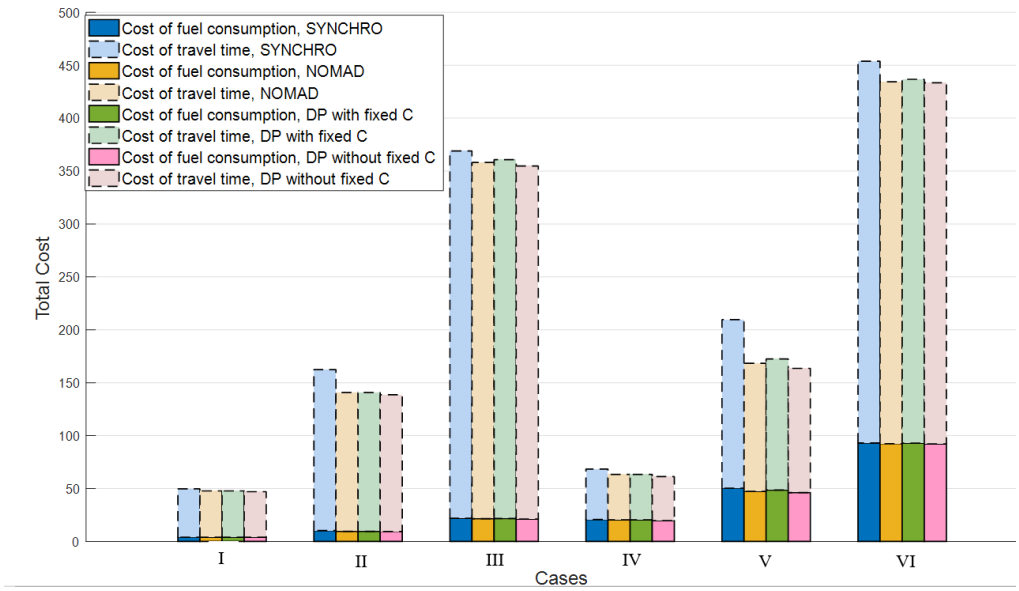
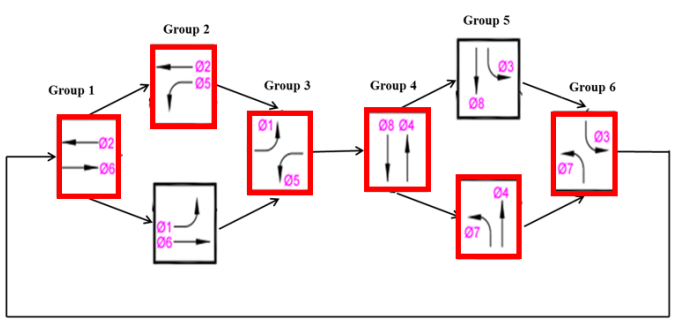


Figure 11: Total cost comparisons

4.1.3 Branch and bound algorithm

This section illustrates how the branch and bound algorithm can be applied to the DP results (without the fixed cycle length constraint) to guarantee a solution with the fixed cycle length. Here we use the DP result of Case I in Table 3. Figure 12 shows the result from the DP by considering the end stage cost, which leads to a cycle length of 57s, 3 seconds lower than the predefined and fixed cycle length of 60s.



x_p	5	11	10	9	11	11	57s < C = 60s
EG (from DP)	0.58	0.93	0.88	0.84	0.94	0.92	TC= 4.76 (using IDM)

Figure 12: Estimated solution from DP

With the information from the DP solutions, the estimated Error Gain (EG) from DP, the total cost of fuel consumption and travel time, and the fixed cycle length, the branching could be selected based on the value of EG. In this case, since phase group 5 (phase 7) has the maximum EG, it is selected as the first level of branching, guided by the algorithm presented in the previous section. Nodes 1 – 3 enumerate each possible value of the decision variable for phase 7 in phase group 5, with the green time for phase 7 being 12, 13, and 14, respectively. Only node 3 is feasible since its phase durations add up to the fixed cycle length 60s. For nodes 1 – 2, the next level of branching is generated by the same rule. Figure 13 is the tree for branch and bound that lists all the feasible solutions with the total cost estimated using IDM. The feasible solution with the minimum objective value (TC) represents the optimal solution. It is observed that the total cost for the optimal solution in Figure 13 is larger than the total cost of the initial solution from DP (C=57s) because we sacrifice the signal performance by adding a fixed cycle length constraint when using the branch and bound method. The signal timing parameter estimation algorithm is as follows:

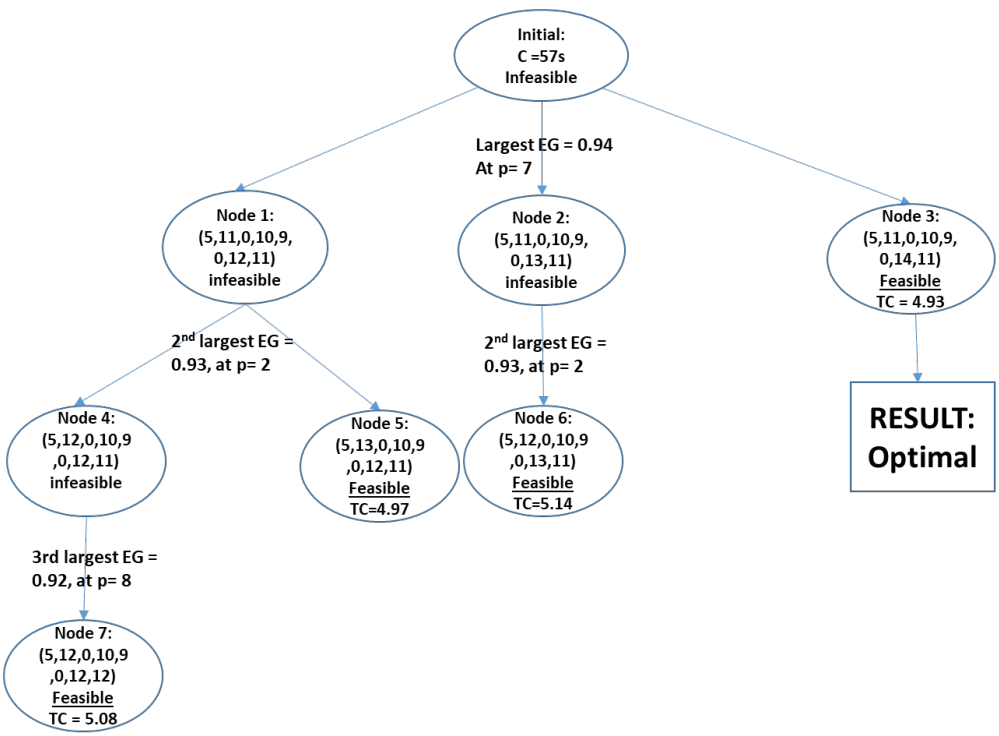


Figure 13: Branch and Bound Tree

4.1.4 Tolerance parameter of branch and bound method

In Eq.(*32*), we define a tolerance σ to calculate the end stage cost to find the DP solution. Different values of σ may produce difference intial solutions and influence the number of evaluations in the branch and bound algorithm. Here an evaluation means that for a given tentative signal timing plan, we need to estimate the objective in Eq. (*25*) using IDM, which may be time consuming. As shown in Figure 13, one node in the graph corresponds to one signal timing plan that needs to be evaluated. We test the tolerance

σ from 0 to 10 and test the performance of the algorithm for Case I. In Figure 14, as σ increases, the total cost decreases slightly and attains its minimum value at $\sigma = 8$, but the number of evalutations increases dramatically. A much larger value of σ (e.g., 10) will need to evalutate more timing plans, but may not help much to minimize the objective. Similar trends can be found for other cases. Seting $\sigma = 5$, as we use in this paper, seems a good balance between solution quality and the computational effort of the algorithm.

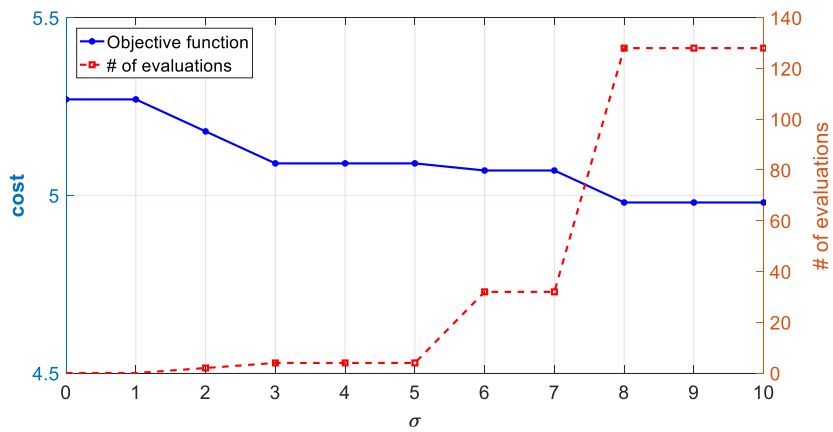


Figure 14: Influence of sigma on the total cost for Case I

4.2 Multiple intersections on a corridor

The proposed CV-based traffic signal optimization and coordination models are tested on a corridor that contains five signalized intersections. The distance between two adjacent intersections is 800 meters (0.5 mile). The West-East direction is the coordinated direction. Each intersection has a boundary of 400m upstream and 400m downstream of the intersection, as shown in Figure 15. It is assumed that upstream infrastructure-based detector data are available for each intersection to provide information on vehicle arrival. In this study, the vehicles are randomly generated at the boundary of the entire network with known arrival times, initial speeds and turning movements. At each intersection, 80% of the vehicles will use the through movement and others will turn left. We also assume that there is only one lane per incoming approach, so no lane changing behavior is modeled. The penetration rate for CV is 100% in this study. We evaluate the proposed signal timing optimization/coordination methods in three steps. First, we estimate the optimal signal timing parameters, including cycle length, phase duration, and offset in SYNCHRO, for different traffic demand levels. Second, for each given scenario that considers different combinations of traffic demands and vehicle types, we apply different methods to optimize signal timing plans (including the phase sequence and durations of each intersection and its offset). Third, we evaluate the performance of each signal timing plan generated from different methods by implementing the plans in the corridor and using IDM to generate vehicle trajectories, based on which the total cost of fuel consumption and travel time is calculated.

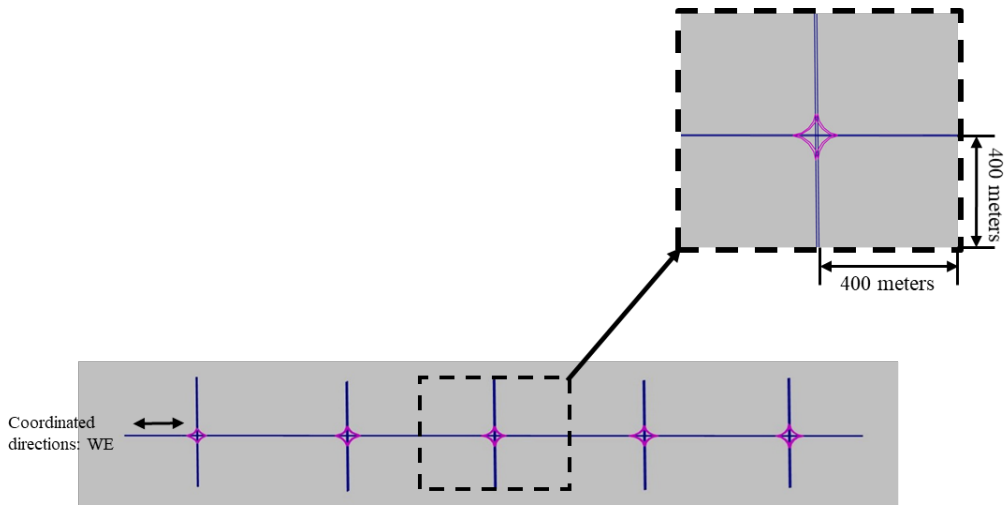


Figure 15: Simulation network containing five intersections

Six cases are tested in order to evaluate the proposed signal optimization methods considering different combinations of traffic demand levels and vehicle types. In Case I – Case III, vehicle demands for the main street (W - E direction) are 250 vph, 500 vph, and 800 vph. For minor streets (N - S direction), the demands are set to be 125 vph, 250 vph, and 250 vph. All vehicles are assumed to be sedans. In Case IV – Case VI, vehicle demand levels are identical to Case I – Case III, but vehicle types are assigned differently. In the N-S directions, vehicles are assigned as Electric Vehicles (EVs), while in the main directions, vehicles are assigned as buses.

Table 5 shows the total costs estimated using different signal plans generated from three methods in 10 cycles. The first method is the actuated signal plan produced from SYNCHRO. Given the geometric information of the corridors and volumes on each movements, SYNCHRO can calculate the optimal signal parameters, including phase durations, cycle length, and offsets. The optimal signal plan, together with the randomly generated vehicle arrival information and updated trajectories using IDM, are implemented in the simulation network to estimate the objective function value, i.e., the total cost of fuel consumption and travel time using Eq. (36-55). The same procedures are applied to the other two methods to calculate the total cost. Note that we consider a fixed cycle length constraint in this study in order to conduct signal coordination. The fixed cycle lengths for different demand levels for all methods are determined by SYNCHRO: they are 60s for low traffic demand (Case I and IV), 80s for medium traffic demand (Case II and V), and 120 for high demand (Case III and VI). The second method, MINLP in Eq. (36-55), is solved by the "NOMAD" solver in Matlab. There are eight phases for each signal, as shown in Figure 5. Considering that we update the signal plan every 10 cycles, there are 40 variables of phase durations and 4 variables of offsets for the simulation network containing five signals. The third method is the decentralized two-level model that can be solved by the proposed prediction-based approach. At the intersection level, the phase durations for each intersection are solved by the DP method and can be updated every cycle. At the corridor level, the offsets are updated every 10 cycles.

In Table 5, the minimal total cost for each case is highlighted. For all cases, the results from MINLP and the two-level model are better that those from SYNCHRO. For Cases III and IV, the results from two-level model perform better than MINLP. Figure 16 shows the improvement of model performance for each case. For Case I and Case IV, with relatively low demand levels, the improvement is smaller than in other cases, which may suggest that coordination has limited benefits when traffic volume is low.

Method	Case I	Case II	Case III	Case IV	Case V	Case VI
	NS: 125vph, WE: 250 vph; All Sedan	NS: 250 vph, WE: 500 vph All Sedan	NS: 250vph,WE: 800 vph All Sedan	NS: 125vph, WE: 250 vph; NS:Evs, WE:Bus	NS: 250 vph, WE: 500 vph; NS:Evs, WE:Bus	NS: 250vph,WE: 800 vph; NS:Evs, WE:Bus
1. SYNCHRO	232.07	804.34	2047.8	320.44	1088.5	2778.5
2. MINLP	227.44	737.53	1998.03	316.86	1036.23	2623.95
3. Two-level model	229.5	764.42	1884.63	314.81	1065.7	2675.47

Table 5: Total cost from different methods under various demand levels and vehicle types

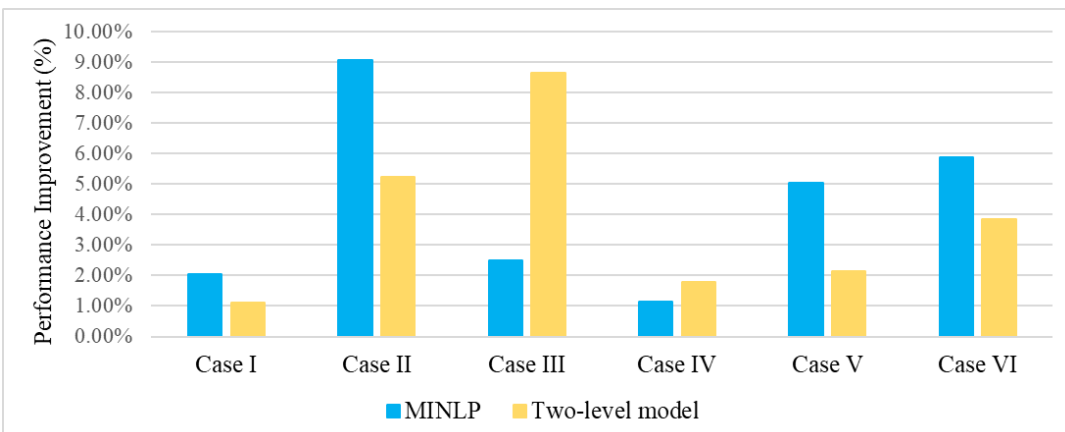


Figure 16: Improvement of model performance over SYNCHRO

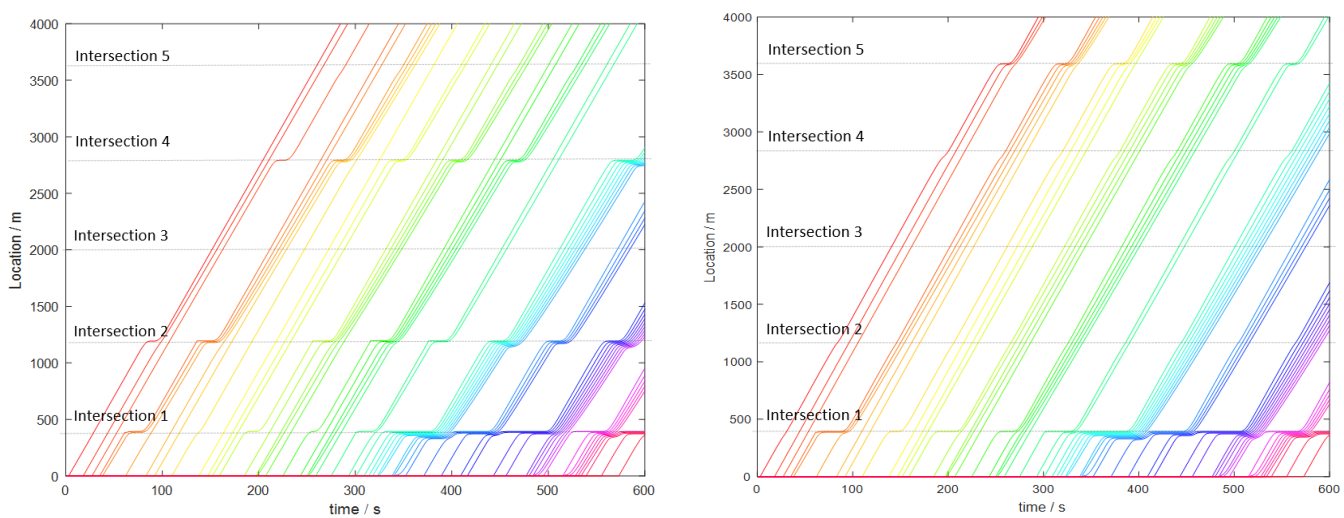
To further evaluate whether coordination can benefit the road network, including both the main street and minor street under different scenarios, we compare the improvement of model performance between two scenarios: with coordination and without coordination (offset = 0). For example, in MINLP, we first solve the model that contains only 40 variables of phase durations and all offset values are set to be zero. We then implement the estimated signal plan in IDM and estimate the total cost for the main street and the minor street separately. These costs are compared with the results from solving the entire model, i.e., Eq. (36-55), which optimizes both phase durations and offsets. The same procedures apply to the two-level model. Table 6 shows the comparison results. The negative values are highlighted in the table, which suggests that by applying coordination, the performance for the main street or the minor street gets worse. For the main street (left number in each cell), MINLP and the two-level model both

underperform under low demand levels (Case I or IV) if applying coordination, while the improvements are more significant for higher demand levels (Cases II, III, V, VI). The results suggest that coordination schemes may not be beneficial to a corridor with low traffic volumes and random arrival vehicles because those vehicles are less likely to form a platoon that will be influenced significantly by the operation of adjacent intersections. For the minor street (right number in each cell), the impacts are relatively small no matter if they are positive or negative. Coordination on the main street seems to have little impact on vehicles on the minor street.

Method	**Case I**	**Case II**	**Case III**	**Case IV**	**Case V**	**Case VI**
	NS: 125vph, WE: 250 vph; All Sedan	NS: 250 vph, WE: 500 vph All Sedan	NS: 250vph,WE: 800 vph All Sedan	NS: 125vph, WE: 250 vph; NS:Evs, WE:Bus	NS: 250 vph, WE: 500 vph; NS:Evs, WE:Bus	NS: 250vph,WE: 800 vph; NS:Evs, WE:Bus
	main street | minor street					
MINLP	**-2.5%** | **-1.5%**	9.1% | **-0.6%**	4.5% | 1.2%	**-2.7%** | **-2.9%**	4.6% | 1.6%	5.9% | 0.3%
Two-level model	**-1.1%** | 0.5%	8.7% | 0.7%	3.7% | **-0.7%**	0.5% | 0.6%	7.8% | **-0.8%**	3.8% | 0.2%

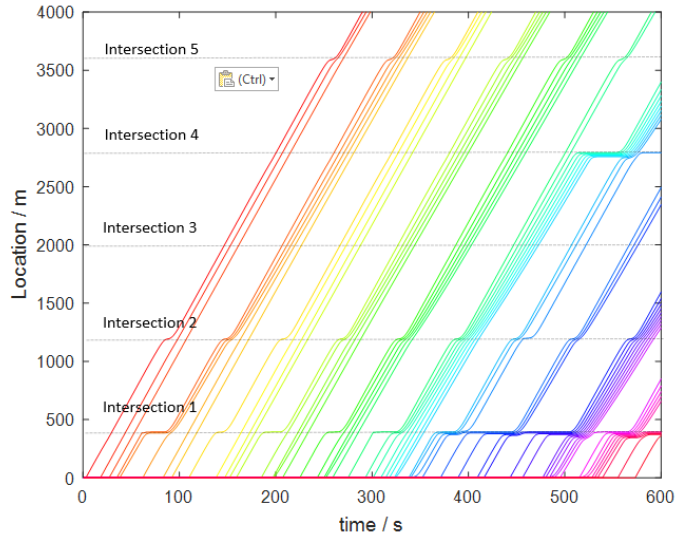
Table 6: Model performance improvement from coordination for main street and minor street

Figure 17 shows vehicle trajectories updated based on different signal plans for 10 cycles (600s) along a 4000m corridor. The vehicles in the figure have coordinated movement (W→E direction in Figure 15) and are randomly generated at the boundary of the corridor. Compared with the trajectories generated using the SYNCHRO plan in Figure 17(a), the delays and number of stops in the other two figures (Figure 17(b) and (c)) are significantly reduced by applying the MINLP method and the two-level model. Through optimization of signal plans, the randomly generated vehicles form vehicle platoons to pass through the intersections smoothly.



(a) Trajectories from SYNCHRO signal plan

(b) Trajectories from MINLP signal plan



(c) Trajectories from two-level model signal plan

Figure 17: Vehicle trajectories from different signal plans

The numerical experiments test different methods in 10 cycles ranging from 10 min to 20 min based on the cycle length. The simulation period can be extended to a longer time, but this requires a significantly longer computation time. It is influenced by several factors, e.g., update intervals of signal plans, whether considering various vehicle types, and the level of traffic demand in the network. For the NOMAD solver, it may fail to find the feasible solutions under high vehicle demand levels and short signal plan updating intervals (meaning larger number of variables). The two-level model can generally ensure convergence, but the computation time also varies based on the aforementioned factors. Figure 18 shows the number of iterations for different offset updating intervals for Case I. Usually, the program will converge within 10 iterations for different cases. Similar patterns are found for other cases, which are omitted here.

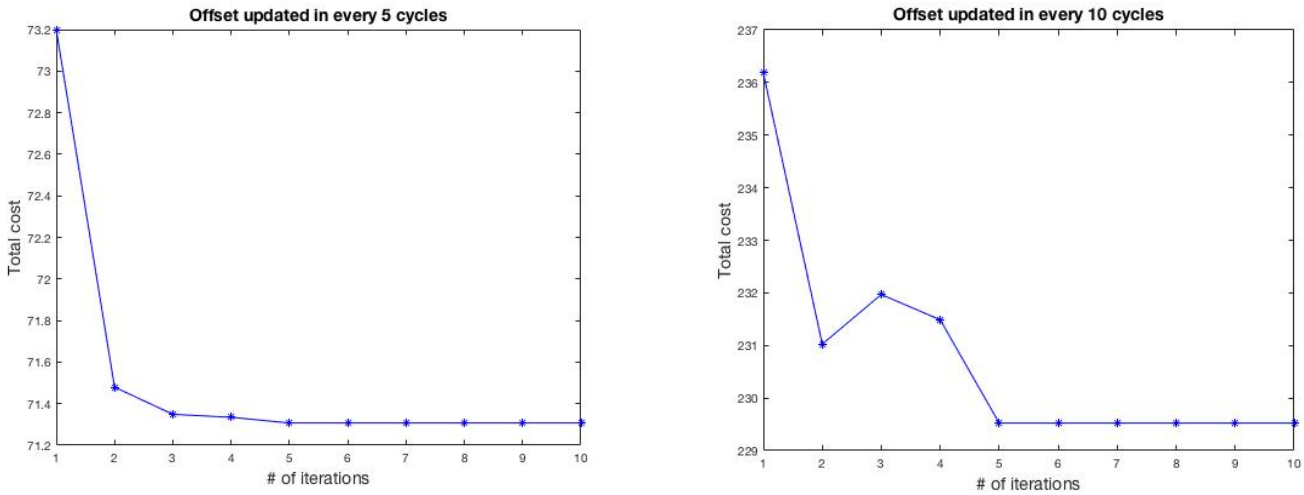


Figure 18: Optimization results of the two-level model for case 1

5. Conclusions

This project presented a signal timing optimization model for a single intersection and along a corridor containing multiple intersections with a fixed cycle length under the CV environment. For a single intersection, the algorithm utilized arrival information (speeds, locations, etc.) from CV as the input to optimize the green times by considering vehicles' fuel consumption and travel time. The problem was first formulated as a MINLP by applying the IDM to predict vehicle trajectories. Such a formulation has a large dimension and a complex car-following model (the IDM). A DP formulation was then developed to approximate the MINLP. The overall problem was divided into stages (one stage for each signal phase). The objective is the summation of the objective of each stage. The objective function of a stage was approximated as a function of the state and decision variables of the stage only, by approximating the vehicle speeds and delays. It is shown that imposing the fixed cycle length constraint would invalidate the DP formulation. Then a two-step method was applied to address this issue. First, an end-stage cost was added to the DP formulation, defined by how much the DP solution violates the fixed cycle length constraint. This step forced the DP to produce a solution with a cycle length that is close to the given fixed cycle length. The second step was a branch and bound method to further refine the DP results to obtain a solution that produces the given cycle length exactly.

For a corridor that contains multiple intersections, the problem also can be formulated in a centralized scheme as a MINLP considering the fixed cycle length constraint to optimize the phase durations and offsets in one mathematical program. IDM was applied to estimate and predict vehicle trajectories considering 100% penetration rate of CVs. Due to the complexity of the model, we decentralized the problem into two levels: an intersection level to generate optimal phase durations using the DP method and a corridor level to update the optimal offsets for all intersections. The two-level model reduces the complexity of the MINLP. In order to solve the two-level model, a prediction-based solution technique was developed that can solve the problem iteratively.

The performance of the algorithm was evaluated using data generated from traffic simulation. For a single intersection, the results of the proposed DP model were compared with two other models. The first one is the traditional actuated signal timing plan generated by SYNCHRO. The second is to solve the MINLP formulation directly using the NOMAD solver in MATLAB. The results showed that the proposed DP method is always superior to SYNCHRO under all cases and can generate similar (slightly worse) solutions compared with NOMAD. However, NOMAD has difficulties finding optimal solutions when the number of variables is relatively large. This makes the proposed DP method more favorable when dealing with large problems (e.g., for multiple cycles). For a corridor, the results from MINLP and the two-level model both outperformed the signal optimization and coordination plan generated by SYNCHRO. This was tested for six cases that consider various combinations of traffic volumes and vehicle types. The results also

suggested that signal coordination may bring limited benefits to intersections with low traffic volumes or to the vehicles on the minor street.

Overall, the solution obtained from the proposed models satisfies the fixed cycle length constraint and ensures a minimum total cost of the weighted sum of fuel consumption and travel time. Future work may also investigate how different penetration levels of CV-equipped vehicles will affect the performance of the proposed signal control method. This will require estimating the trajectories of vehicles that are not equipped with CV technology. When sample trajectory data from the real world are available, certain stochastic models, e.g., Kalman filter based or Bayesian methods, may be applied to estimate and predict the trajectories. For this, past work of the authors[55] on estimating vehicle trajectories at signalized intersections may be helpful. Furthermore, the proposed method needs to be tested using real world traffic signals and CV data. This will be pursued in future work.

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