RESILIENT BUS SERVICES DESIGN UNDER CORRELATED STOCHASTIC METRO SYSTEM DISRUPTION

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Abstract: This study develops a robust multimodal network design integrating metro and bus services, addressing correlated stochastic disruptions. Using a vine copula technique, we model dependencies among disrupted metro links for realistic correlation representation. A two-stage correlation-aware stochastic program is formulated to optimize bus routing and frequencies, minimizing total costs while accounting for unmet demand. By designing the entire network to withstand metro disruptions, the proposed approach demonstrates superior performance, reducing demand losses by 99% compared to benchmark that neglects correlation. This framework enhances the resilience of public transport systems under uncertainty, ensuring efficient and reliable service during disruptions.

Keywords: network design; metro disruption; multimodal network; reliability; optimization under uncertainty

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1. Introduction

Metro and bus systems are the backbone of urban transportation, yet metro systems are highly vulnerable to disruptions, which can cascade across the network, causing widespread delays and decreased passenger satisfaction (Zhang et al., 2021). Existing research on disruption management can be broadly categorized into two areas. The first focuses on post-disruption responses, e.g., optimizing bus bridging services (Jin et al., 2016), while the second emphasizes pre-disruption planning to enhance system resilience, e.g., (Jin et al., 2014) proposed leveraging existing bus services to improve metro network resilience. Most studies did not consider the potential of regular bus services as a complementary solution to metro systems, particularly under stochastic disruptions. Hence, we propose a transit network design that integrates buses as a complementary mode to enhance system resilience, balancing service cost with adaptive capacity during disruptions.

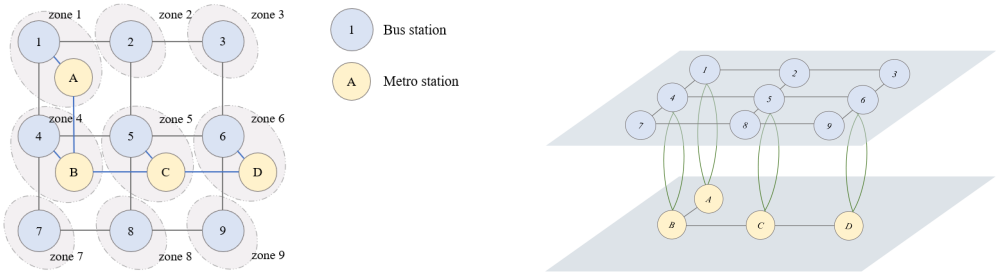
While disruptions are commonly treated as independent events, it is important to recognize that they can have a ripple effect, underscoring the need to model their interdependencies. However, modeling correlated

distributions of metro line disruptions is also overlooked by existing research. Standard multivariate distributions can become inflexible in high dimensions and do not allow for different dependency structures between pairs of variables, especially in our framework of understanding cascading failures in transit networks. Vine copulas, first introduced in Bedford and Cooke (2002), offer a flexible approach to model high dimensional dependency. For example, Wissink (2023) demonstrated the utility of pair copulas in modeling stochastic dependencies in routing problems. Inspired by these advantages, we leverage vine copulas to capture the stochastic interdependencies of metro disruptions, laying a foundation for a more realistic and resilient network design.

This study presents a two-stage correlation-aware stochastic programming approach for designing multimodal transit networks. In the first stage, bus routes and frequencies are optimized to minimize construction, operational, and link overlap penalty costs, as well as the expected costs of the second stage. In the second stage, under correlated metro disruption scenarios, passenger flows are assigned to minimize passenger and unmet demand costs. A case study applies the decoupled service reliability (d-SR) method, ensuring computational efficiency and robust network performance. Results show a significant reduction in unmet demand, up to 99%, compared to benchmarks that assume independent disruptions on vulnerable links.

2. Model Formulation

2.1 Metro-Bus Multimodal Network



(a) Zonal-based representation **(b) Bus layer and metro layer**

Figure 1: Graphical illustration of the sample multimodal network

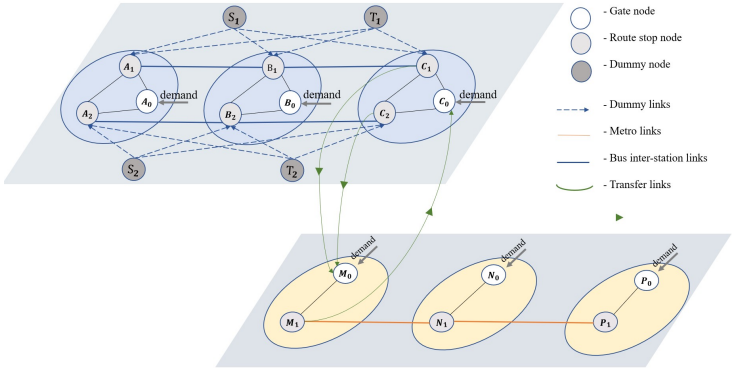


Figure 2: A detailed network for two bus routes and one metro route

We consider a bus network design problem alongside an existing metro line prone to capacity degradation. A detailed representation of the network will be provided in this section for the bus network design problem. Demand is clustered on a zone-to-zone basis, with zones containing bus and/or metro stations. Each station belongs to a specific zone, and bus and metro services operate between zones with one stop per zone. In case of metro disruptions, buses serve as alternatives for unmet demand. Figure 1(a) shows the zonal network,

where zones handle demand collaboratively. Figure 1(b) separates bus and metro layers, with intermodal transfer links connecting services within the same zone. Figure 2 illustrates the multimodal network with three zones. The upper layer represents the bus network, and the lower layer represents the metro network. Stations C and M form a hybrid zone, while stations A, B, N, and P are bus-only or metro-only zones, respectively. Stations are decomposed into gate nodes and route stop nodes. For example, bus stations with two routes have one gate node and two route stop nodes, linked via in-station connections. The bus network includes dummy origins and destinations. Metro stations include a gate node and one route stop node. Solid lines indicate bidirectional links; arrows represent directed links.

2.2 Modeling Link Dependence of Metro Line

Denote F as the joint distribution of link capacity of the metro line $(\Delta_1, \ldots, \Delta_n)$. Sklar's theorem states that every multivariate distribution F with marginals $F_1(x_1), \ldots, F_n(x_n)$ can be written as $F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n))$, where $C : [0,1]^n \mapsto [0,1]$ is a copula function modeling the dependence structure of the stochasticity. For continuous random variables, we can further derive the joint density $f(x_1, \ldots, x_n)$ in terms of copula density $c(u_1, \cdots, u_n) \coloneqq \partial C / \partial u_1 \cdots \partial u_n$ and marginal densities $f_1(x_1), \ldots, f_n(x_n)$ by $f(x_1, \ldots, x_n) = c(F_1(x_1), \ldots, F_n(x_n)) \prod_{k=1}^n f_k(x_k)$. The standard parametric multivariate copulas are more challenging in high dimensions. For example, the elliptical copulas, including Gaussian copula and t-copula are restricted to symmetric dependence modeling. To allow for modeling different dependence between pairs of variables, Aas et al. (2009) show the vine copula construction methods to construct high dimension copulas using bivariate copulas as building blocks. We study their methods to model the link dependence of the metro lines.

2.2.1 A Simpler Model: Neighboring Link Dependence (NLD)

Definition 1 (Neighboring Link Dependence) *An n-link metro line has Neighboring Link Dependence (NLD) if the joint distribution can be represented as*

$$f(x_1, \ldots, x_n) = \prod_{i=1}^{n-1} c_{i,i+1}(F_i(x_i), F_{i+1}(x_{i+1}); \theta_{i,i+1}) \prod_{k=1}^{n} f_k(x_k), \tag{1}$$

i.e., the copula density is $c(u_1, \ldots, u_n) = \prod_{i=1}^{n-1} c_{i,i+1}(u_i, u_{i+1}; \theta_{i,i+1})$ *where* $\theta_{i,i+1}$ *is the parameter.*

Note that each pair of adjacent links can have a different copula structure, offering a more complex dependence framework than is typical with standard parametric copulas. The NLD model demonstrates a conditional independent property, as shown in Proposition 1.

Proposition 1 *If an n-link metro line has NLD, then for any two links* i, j *such that* $i < j+1$*, conditional on link capacity of* $\Delta_{i+1}, \ldots, \Delta_{j-1}$*, the link capacities* Δ_i *and* Δ_j *are independent.*

Proof of Proposition 1. For any continuous n-dimensional density, we use D-vine to factorize it as

$$f(x_1, \ldots, x_n) = \prod_{j=1}^{n-1} \prod_{i=1}^{n-j} c_{i,i+j|\mathscr{S}_{i,i+j}}(F(x_i|\mathscr{S}_{i,i+j}), F(x_{i+j}|\mathscr{S}_{i,i+j})) \times \prod_{k=1}^{n} f_k(x_k).$$

If the dependence is nearest-neighbored, then we also have

$$f(x_1, \ldots, x_n) = \prod_{i=1}^{n-1} c_{i,i+1}(F_i(x_i), F_{i+1}(x_{i+1})) \prod_{k=1}^{n} f_k(x_k).$$

Comparing these two factorizations, we immediately conclude that for nearest-neighbor dependency, we have,

$$c_{i,i+j|\mathscr{S}_{i,i+j}}(F(x_i|\mathscr{S}_{i,i+j}), F(x_{i+j}|\mathscr{S}_{i,i+j})) \equiv 1 \quad \text{for any } j > 1,$$

which is equivalent to say

$$c_{i,j|\Delta_{i+1},\ldots,\Delta_{j-1}}\left(F(x_i|\Delta_{i+1},\ldots,\Delta_{j-1}),F(x_j|\Delta_{i+1},\ldots,\Delta_{j-1})\right)\equiv 1 \quad \text{for any } j>i+1.$$

Then we have

$$F(x_i,x_j|\Delta_{i+1},\ldots,\Delta_{j-1})=F(x_i|\Delta_{i+1},\ldots,\Delta_{j-1})F(x_j|\Delta_{i+1},\ldots,\Delta_{j-1}),$$

i.e. the conditional independence of Δ_i and Δ_j. The proof is completed.

2.2.2 Vine-Based Dependence Model

The vine model (Bedford and Cooke, 2002) captures additional conditional dependencies by incorporating conditional bivariate copulas into high-dimensional copula constructions. Using Bayes' theorem, the joint density of link capacities of a metro line can be expressed as:

$$f(x_1,\ldots,x_n)=f_n(x_n)f(x_{n-1}|x_n)\cdots f(x_1|x_2,\ldots,x_{n-1}). \tag{2}$$

Each conditional density in Equation (2) can be further factorized as a conditional bivariate copula and a conditional marginal density by invoking: $f(x_i|\boldsymbol{v})=c_{i,j|\boldsymbol{v}_{-j}}(F(x_i|\boldsymbol{v}_{-j}),F(v_j|\boldsymbol{v}_{-j}))$, where $\boldsymbol{v}$ is any vector, v_j is an arbitrarily chosen element of $\boldsymbol{v}$, and $\boldsymbol{v}_{-j}$ represents the vector of $\boldsymbol{v}$ excluding element v_j. By iteratively factorizing the conditional probabilities, the joint density can be expressed as a product of conditional bivariate copulas and marginal densities $\prod_{k=1}^{n} f_k(x_k)$. We can thus parameterize each conditional bivariate copula. Different ways of factorization yield different vine classes. Below we present the D-vine-based dependence.

Definition 2 (D-vine-based dependence) *An n-link metro line has D-vine-based dependence if*

$$f(x_1,\ldots,x_n)=\prod_{j=1}^{n-1}\prod_{i=1}^{n-j} c_{i,i+j|\mathscr{S}_{i,i+j}}\left(F(x_i|\mathscr{S}_{i,i+j}),F(x_{i+j}|\mathscr{S}_{i,i+j});\theta_{i,i+j|i+1,\ldots,i+j-1}\right)\times\prod_{k=1}^{n} f_k(x_k). \tag{3}$$

where $\theta_{i,i+j|i+1,\ldots,i+j-1}$ parameterizes the conditional bivariate copula and $\mathscr{S}_{i,i+j}=\{x_{i+1},\ldots,x_{i+j-1}\}$ denotes the set of conditional variables. The conditioning set is dropped when $\mathscr{S}_{i,i+j}=\emptyset$.

The vine-based dependence model can capture a wide range of stochastic capacity distributions and their dependencies. While more complex than the NLD model, it preserves the conditional dependence of each link pair. For links that are distant within the network and assumed to be independent, the corresponding conditional bivariate copula density can be set to a constant value of 1, reducing the number of parameters. NLD is a special case of D-vine-based Dependence.

2.3. Two-Stage Correlation-Aware Stochastic Programming

We first introduce some notation and define the cost terms and constraints. Disruption scenarios are represented by $\Xi=\{1,2,\ldots,N\}$, each with equal probability $p_\xi=\frac{1}{N}$, where $\xi\in\Xi$ and $\sum_{\xi\in\Xi}p_\xi=1$. Scenario-specific variables are denoted by the superscript $(\cdot)^\xi$. Each scenario is a realization of stochastic metro link capacities sampled from the joint distribution with correlation.

We aim to minimize the sum of all costs expressed below.

Bus construction and operating cost: $\sum_{r\in R_b}\sum_{ij\in A^b}c^1_{ij}y^r_{ij}+\sum_{ij\in A^b}c^2_{ij}\bar{Y}_{ij}+\sum_{i\in N^b}c^3W_i.$

Passenger cost: $\sum_{ij\in A^b}\sum_{d\in D}c^4_{ij}X^d_{\text{in-vehicle}}+c^5\sum_{r\in R_b}\sum_{d\in D}\left(\frac{1}{2f_r}X^d_{\text{board}}+\left(\tau+\frac{1}{2f_r}\right)X^d_{\text{transfer}}\right)+c^6\sum_{d\in D}V^d_{\text{bus-metro}}$
$+c^6\sum_{d\in D}V^d_{\text{metro-bus}}+\sum_{ij\in A^m}\sum_{d\in D}c^7_{ij}H^d_{\text{in-vehicle}}.$

Overlapping penalty: $\sum_{r\in R_b}\sum_{ij\in A^b}c^8_{ij}O_{ij}\cdot Y^r_{ij}.$

Table 1: Multimodal Network Parameters

N, N^b, N^m	Station set, bus station set, metro station set
A, A^b, A^m	Link set, Bus link set, Metro link set
$A^{\text{bis}}, A^{\text{mis}}, A^{\text{rmis}}$	Bus inter-station link set, metro inter-station link set, relabelled Metro inter-station link set
$A^{\text{it}}, A^{\text{vul}}$	Intermodal transfer link set, vulnerable metro link set subject to stochastic disruptions
Δ	Metro link capacity (*a multivariate random variable*)
$G(N,A)$	Transportation network with node set N and link set A
$(i,j) \in A$	Feasible link from node i to $j, i, j \in N^b$ or $i, j \in N^m$
R_b	Bus route set
i_0, i_r	Gate sub-nodes of station i, route sub-nodes of station i
$S = \{S_r\}$	Dummy origin set, $r \in R_b$
$T = \{T_r\}$	Dummy destination set, $r \in R_b$
S_r, T_r	Dummy origin and dummy destination nodes of route $r, r \in R_b$
l_{ij}	Link distance from station i to station j, $ij \in A^{\text{bis}}$ or $ij \in A^{\text{mis}}$
$g_d^{m+} \in N^m$, $g_d^{m-} \in N^m$	Metro station in the origin zone of $d \in D$, metro station in the destination zone of $d \in D$
$g_d^{b+} \in N^b$, $g_d^{b-} \in N^b$	Bus station in the origin zone of $d \in D$, bus station in the destination zone of $d \in D$

Table 2: Network Link Types

(i_u, i_v)	$\forall i \in N^b, u, v \in R_b \cup \{0\}, u \neq v$	General representation for bus in-station links
(i_0, i_r)	$\forall i \in N^b, r \in R_b$	Boarding passengers entering the bus station i for bus route r
(i_r, i_0)	$\forall i \in N^b, r \in R_b$	Alighting passengers leaving the bus station i from bus route r
(i_u, i_v)	$\forall i \in N^b, u, v \in R_b, u \neq v$	Transfer links between different bus routes within the bus station i
(i_r, j_r)	$\forall (i,j) \in A^{\text{bis}}, r \in R_b$	Links between bus stations i and j using bus route r
(S_r, i_r)	$\forall i \in N^b, r \in R_b$	Dummy links representing start stops of each bus route r
(i_r, T_r)	$\forall i \in N^b, r \in R_b$	Dummy links representing end stops of each bus route r
(i_r, j_0)	$\forall (i,j) \in A^{\text{it}}, r \in R_b$	Links between bus route sub-nodes and metro gate sub-nodes
(j_1, i_0)	$\forall (j,i) \in A^{\text{it}}$	Links between metro route sub-nodes and bus gate sub-nodes
(i_u, i_v)	$\forall i \in N^m, u, v \in \{0,1\}, u \neq v$	General representation for metro in-station links
(i_0, i_1)	$\forall i \in N^m$	Boarding passengers entering the metro station i for metro line
(i_1, i_0)	$\forall i \in N^m$	Alighting passengers leaving the metro station i from metro line
(i_1, j_1)	$\forall (i,j) \in A^{\text{rmis}}$	Links between metro stations i and j using metro line

Table 3: Decision Variables

$\mathbf{Y} = \{Y_{ij}^r\}$	Binary variable indicating inclusion of inter-station link (i_r, j_r) in bus route r
$\bar{\mathbf{Y}} = \{\bar{Y}_{ij}\}$	Binary variable indicating if link (i,j) is covered by any bus route
$\mathbf{W} = \{W_i\}$	Binary variable indicating if a station has at least one bus route passing through it
$\mathbf{f} = \{f_r\}$	Frequency set for each bus route r
$\mathbf{y} = \{y_{ij}^r\}$	Effective frequency, product of Y and f, $y_{ij}^r = Y_{ij}^r f_r$
$h = \{h_i^r\}$	Intermediate variable representing the order of bus route r at station i
$B_b = \{B_{b,i}^d\}$	Demand variables for the bus system on station i for OD pair d
$B_m = \{B_{m,i}^d\}$	Demand variables for the metro system on station i for OD pair d
$X_{\text{board}}^d = \{X_{i_0 i_v}^d\}, v \in R_b$	Boarding passenger flow to bus route v at station i.
$X_{\text{alight}}^d = \{X_{i_u i_0}^d\}, u \in R_b$	Alighting passenger flow from bus route u at station i.
$X_{\text{transfer}}^d = \{X_{i_u i_v}^d\}, u, v \in R_b, u \neq v$	Transfer passenger flow from route u to route v at station i.
$X_{\text{in-vehicle}}^d = \{X_{i_r j_r}^d\}, i \neq j, r \in R_b$	Passenger flow traveling from station i to station j on route r.
$H_{\text{board}}^d = \{H_{i_0 i_1}^d\}$	Boarding passenger flow at station i.
$H_{\text{alight}}^d = \{H_{i_1 i_0}^d\}$	Alighting passenger flow at station i.
$H_{\text{in-vehicle}}^d = \{H_{i_1 j_1}^d\}, i \neq j$	Passenger flow traveling on metro from i to j.
$V_{\text{bus-metro}}^d = \{V_{i_1 j_0}^d\}$	Transfer flows from bus to metro
$\mathbf{Z} = \{Z^d\}$	lost unmet passenger flows for OD pair d

The optimization is subjected to the following constraints.

Lost demand cost: $\sum_{d\in D} c_d^9 Z^{d,\xi}$.

Bus network construction constraints:

$$\sum_{j\in N^b\cup T} Y_{ij}^r \le 1, \quad i\in N^b\cup S; \sum_{j\in N^b\cup S} Y_{ji}^r \le 1, \quad i\in N^b\cup T, \quad \forall r\in R_b, \tag{4}$$

$$\sum_{i\in N^b\cup S} Y_{ik}^r = \sum_{j\in N^b UT} Y_{kj}^r, \quad \forall r\in R_b, k\in N^b \tag{5}$$

$$Y_{ij}^r + Y_{ji}^r \le 1, \quad h_i^r - h_j^r + M\cdot Y_{ij}^r \le M-1, \quad \forall ij\in A^b, r\in R_b \tag{6}$$

$$\bar{Y}_{ij} \le \sum_{r\in R_b}\left(Y_{ij}^r + Y_{ji}^r\right), \quad \bar{Y}_{ij} \ge Y_{ij}^r + Y_{ji}^r, \quad \forall r\in R_b, \forall ij\in A^b, i<j \tag{7}$$

$$f_{\min} \le f_r \le f_{\max} \quad \forall r\in R_b \tag{8}$$

$$y_{ij}^r - f_r \le 0, \quad y_{ij}^r - MY_{ij}^r \le 0, \quad M\left(Y_{ij}^r - 1\right) - y_{ij}^r + f_r \le 0, \quad \forall ij\in A^b, \quad r\in R_b \tag{9}$$

$$W_i \ge \bar{Y}_{ij}, \quad W_i \ge \bar{Y}_{ji}, \quad Y_{ij}^r, \bar{Y}_{ij}, W_i \in \{0,1\}, \quad \forall i\in N^b, j\in N^b, \forall r\in R_b, \quad ij\in A^b \tag{10}$$

Flow balance constraints:

Demand split:

$$\begin{cases} B_{b,g_d^{m+}}^{d,\xi} + B_{b,g_d^{b+}}^{d,\xi} + Z^{d,\xi} = B^{d,\xi} \\ B_{b,g_d^{m-}}^{d,\xi} + B_{b,g_d^{b-}}^{d,\xi} + Z^{d,\xi} = B^{d,\xi} \end{cases} \quad \forall \xi\in\Xi, d\in D \tag{11}$$

Station sub-nodes passenger flow balance:

(a) for each bus station $i\in N^b$, $d\in D, \xi\in\Xi$,

$$\sum_{r\in R_b} X_{i_0 i_r}^{d,\xi} - \sum_{r\in R_b} X_{i_r i_0}^{d,\xi} = \begin{cases} B_{b,i}^{d,\xi} + \sum\limits_{ij\in A^{it}} V_{j_1 i_0}^{d,\xi}, & \text{if } i \text{ is origin of } d \\ -B_{b,i}^{d,\xi} + \sum\limits_{ij\in A^{it}} V_{j_1 i_0}^{d,\xi}, & \text{if } i \text{ is dest. of } d \\ \sum\limits_{ij\in A^{it}} V_{j_1 i_0}^{d,\xi}, & \text{otherwise} \end{cases} \tag{12}$$

(b) for each metro station $i\in N^m$, $d\in D, \xi\in\Xi$,

$$H_{i_0 i_1}^{d,\xi} - H_{i_1 i_0}^{d,\xi} = \begin{cases} B_{m,i}^{d,\xi} + \sum\limits_{ij\in A^{it}}\sum\limits_{r\in R_b} V_{j_r i_0}^{d,\xi}, & \text{if } i \text{ is the origin of } d \\ -B_{m,i}^{d,\xi} + \sum\limits_{ij\in A^{it}}\sum\limits_{r\in R_b} V_{j_r i_0}^{d,\xi}, & \text{if } i \text{ is the dest. of } d \\ \sum\limits_{ij\in A^{it}}\sum\limits_{r\in R_b} V_{j_r i_0}^{d,\xi}, & \text{otherwise} \end{cases} \tag{13}$$

Route sub-nodes passenger flow balance:

(a) for each bus station $i\in N^b$, $r\in R_b, d\in D, \xi\in\Xi$,

$$\sum_{\substack{u\in R_b\cup\{0\}\\ r\neq u}} X_{i_r i_u}^{d,\xi} - \sum_{\substack{u\in R_b\cup\{0\}\\ r\neq u}} X_{i_u i_r}^{d,\xi} + \sum_{ij\in A^{\text{bis}}} X_{i_r j_r}^{d,\xi} - \sum_{ij\in A^{\text{bis}}} X_{j_r i_r}^{d,\xi} + \sum_{ik\in A^{\text{it}}} V_{i_r k_0}^{d,\xi} = 0 \tag{14}$$

(b) for each metro station $i\in N^m$, $r\in R_m, d\in D, \xi\in\Xi$,

$$H_{i_1 i_0}^{d,\xi} - H_{i_0 i_1}^{d,\xi} + \sum_{ij\in A^{\text{mis}}} H_{i_1 j_1}^{d,\xi} - \sum_{ij\in A^{\text{mis}}} H_{j_1 i_1}^{d,\xi} + \sum_{ik\in A^{\text{it}}} V_{i_1 k_0}^{d,\xi} = 0 \tag{15}$$

If the station is non-transfer, the corresponding $\sum_{ij\in A^{it}}\sum_{r\in R_b} V^{d,\xi}_{j_r i_0}$, $\sum_{ij\in A^{it}} V^{d,\xi}_{j_1 i_0}$, $\sum_{ik\in A^{\text{it}}} V^{d,\xi}_{i_r k_0}$, $\sum_{ik\in A^{\text{it}}} V^{d,\xi}_{i_1 k_0}$ will be zero.

Capacity constraints:

(a) for bus bus link capacity:
$$\sum_{d\in D} X^{d,\xi}_{i_r j_r} \le C_{bus}\left(Y^r_{ij}+Y^r_{ji}\right) f_r \quad \forall ij\in A^{\text{bis}}, \quad r\in R_b \tag{16}$$

(b) for metro (vulnerable) link capacity:
$$\sum_{d\in D} H^{d,\xi}_i \le \begin{cases} C_{\text{metro},i}, & \forall i\in A^{\text{rmis}}\setminus A^{\text{vul}} \\ \Delta^{\xi}_i, & \forall i\in A^{\text{vul}} \end{cases} \tag{17}$$

where vulnerable link capacities Δ^{ξ}_i are sampled from the joint distribution of stochastic metro capacities.

Having defined the cost terms and constraints, we now introduce the objective functions for each stage of the analysis. The two-stage stochastic formulation under system optimization (SO) is expressed as follows:

stage 1:
$$\min_{\substack{\boldsymbol{f},\boldsymbol{Y},\boldsymbol{y},\boldsymbol{Y},\\ \boldsymbol{W},\boldsymbol{X},\boldsymbol{V},\boldsymbol{H},\boldsymbol{Z}}} \sum_{r\in R_b}\sum_{i\in A^b} c^1_{ij} y^r_{ij} + \sum_{ij\in A^b} c^2_{ij}\bar{Y}_{ij} + \sum_{i\in N^b} c^3 W_i + \sum_{r\in R_b}\sum_{ij\in A^b} c^8_{ij} O_{ij}\cdot Y^r_{ij} + \bar{L}(\boldsymbol{Y},\boldsymbol{f}) \tag{18}$$

subject to: (4)-(10).

stage 2:
$$\begin{aligned}\bar{L}(\boldsymbol{Y},\boldsymbol{f}) = \sum_{\xi\in\Xi} p_\xi \Bigg[\min_{\boldsymbol{X}^\xi,\boldsymbol{V}^\xi,\boldsymbol{H}^\xi,\boldsymbol{Z}^\xi} \Bigg(& \sum_{d\in D} c^9_d Z^{d,\xi} + \sum_{ij\in A^b}\sum_{d\in D} c^4_{ij} X^d_{\text{in-vehicle}} + c^5 \sum_{r\in R_b}\sum_{d\in D} \frac{1}{2f_r} X^d_{\text{board}} \\ & + c^5 \sum_{r\in R_b}\sum_{d\in D}\left(\tau+\frac{1}{2f_v}\right) X^d_{\text{transfer}} + c^6 \sum_{d\in D} V^d_{\text{bus-metro}} + c^6 \sum_{d\in D} V^d_{\text{metro-bus}} + \sum_{ij\in A^m}\sum_{d\in D} c^7_{ij} H^d_{\text{in-vehicle}} \Bigg) \Bigg]\end{aligned} \tag{19}$$

subject to: (11)-(17). In Eq.(17), the realization of stochastic disruption in stage 2 involves capturing the correlation among links and determine the degraded capacities resulting from metro disruptions by sampling $(\Delta^{\xi}_i,\ldots,\Delta^{\xi}_{|A|^{vul}})$ from the joint distribution of link capacity F. Note that stage 2 is an expectation of the optimal solution of a linear programming as traffic assignment over random scenarios.

To solve above two-stage stochastic program with correlation, we proposed a decoupled Service Reliability (d-SR) solution method as a generalization of the conventional SR method (Lo et al., 2013). The d-SR method performs gradient descent-based optimization on a decoupled reliability space that transforms the correlated reliability to be independent such that optimization is more efficient. After each step of update, the method "re-coupled" the reliability to do the exact objective evaluation. Local optimal solutions can be reached within a finite step of iterations and different initial values can be explored to enhance the solution quality.

3. Case Study

We augment the Nguyen-Dupuis (N-D) network by an additional metro route, as shown in Fig.3(a), thus creating a multimodal transportation system. We consider the metro line running from station 14 to station 18 with vulnerable metro links (14,15), (15,16), (16,17), (17,18), subject to stochastic disruptions with Uniform$(0,200)$ marginal distribution. The correlation is modeled by NLD, which sequentially applies the Clayton ($\theta=6$), the Gaussian ($\theta=0.95$), and the Gumbel ($\theta=6$). The demand volume at OD pair is {(6, 7): 150, (4, 2): 75, (1, 10): 125, (9, 6): 150, (5, 8): 125, (13, 1): 100}. The model parameters are given by $c_1=2, c_2=10, c3=1, c_4=0.05, c_5=0.1, \tau=1, c_6=0.1, c_7=0.005, c_8=1, c_9=5, C_{\text{bus}}=40, m=500, \varepsilon=0.001, k_{\max}=30, f_{\min}=1, f_{\max}=20, \delta=0.005, C_{\text{metro}}=200$. We solve it by d-SR method with

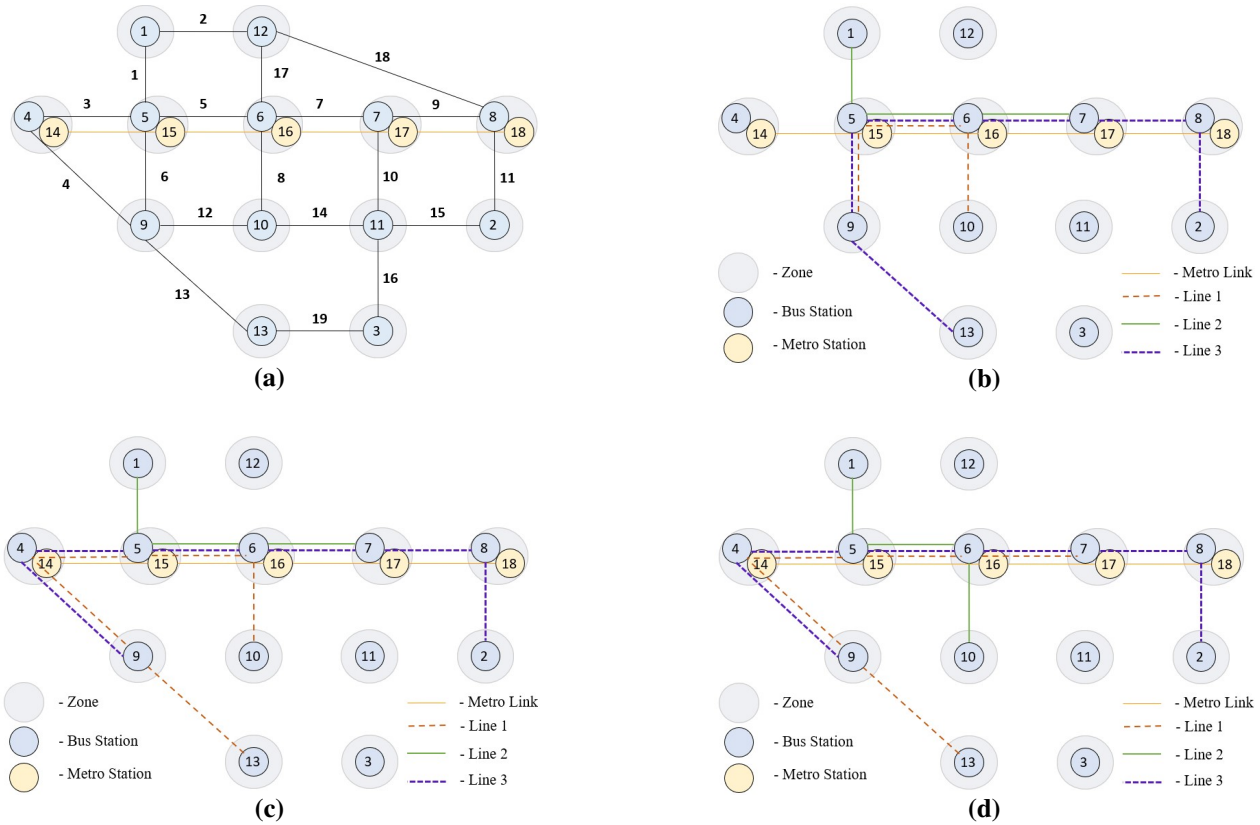


Figure 3: Resultant bus routes. (a): N-D network. (b): Benchmark solution using SR method without considering correlation. (c): d-SR solution with initial reliabilities $0.2, 0.4, 0.6$**. (d): d-SR with initial reliability** 0.8**.**

Table 4: Solutions under d-SR methods and SR benchmark

Method	J	$\bar{L}$	Lost demand	Total cost	f	$\sigma_{\bar{L}}$
d-SR(0.2)	894.60	369.48	0.03	1264.08	(1.250, 1.658, 1.042)	35.11
d-SR(0.4)	890.97	373.47	1.20	1264.44	(1.046, 1.248, 1.608)	37.55
d-SR(0.6)	894.42	369.82	0.08	1264.84	(1.042, 1.243, 1.658)	35.29
d-SR(0.8)	888.67	370.91	11.63	1259.57	(1.034, 1.042, 1.658)	56.67
SR	839.78	459.31	97.67	1299.09	(1.083, 1.908, 1.000)	130.07

different initial values and conventional SR method without correlation consideration as benchmark. The resultant bus routes and costs under different methods are given in Fig. 3(b)-(d) and Table 4 respectively.

Results in Table 4 show the d-SR method significantly reduces expected lost demand (as low as 0.03 vs. 97.67 for the benchmark). The d-SR method also lowers average passenger costs and cost variability, enhancing stability. By accounting for disruption correlations, it designs bus routes that strategically support critical metro links, unlike the SR benchmark, which neglects such interdependencies. Overall, the d-SR method outperforms the conventional SR by minimizing lost demand and improving network resilience, demonstrating its effectiveness in adapting to metro capacity fluctuations while maintaining cost efficiency.

4. Conclusions

This paper presents a new framework to enhance transportation network resilience by addressing correlated disruptions and optimizing network design. We introduce correlation modeling and correlation-aware stochastic programming, improving disruption impact assessments. A case study demonstrates that the new

model significantly reduces lost demand costs and lowers passenger costs. The optimal bus routes provide additional parallel routes to support vulnerable metro links, improving service stability. These findings highlight the effectiveness of correlation modeling in enhancing network resilience. Future work could incorporate demand uncertainties into the model, further strengthening the resilience of the transportation network under a wider range of conditions.

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